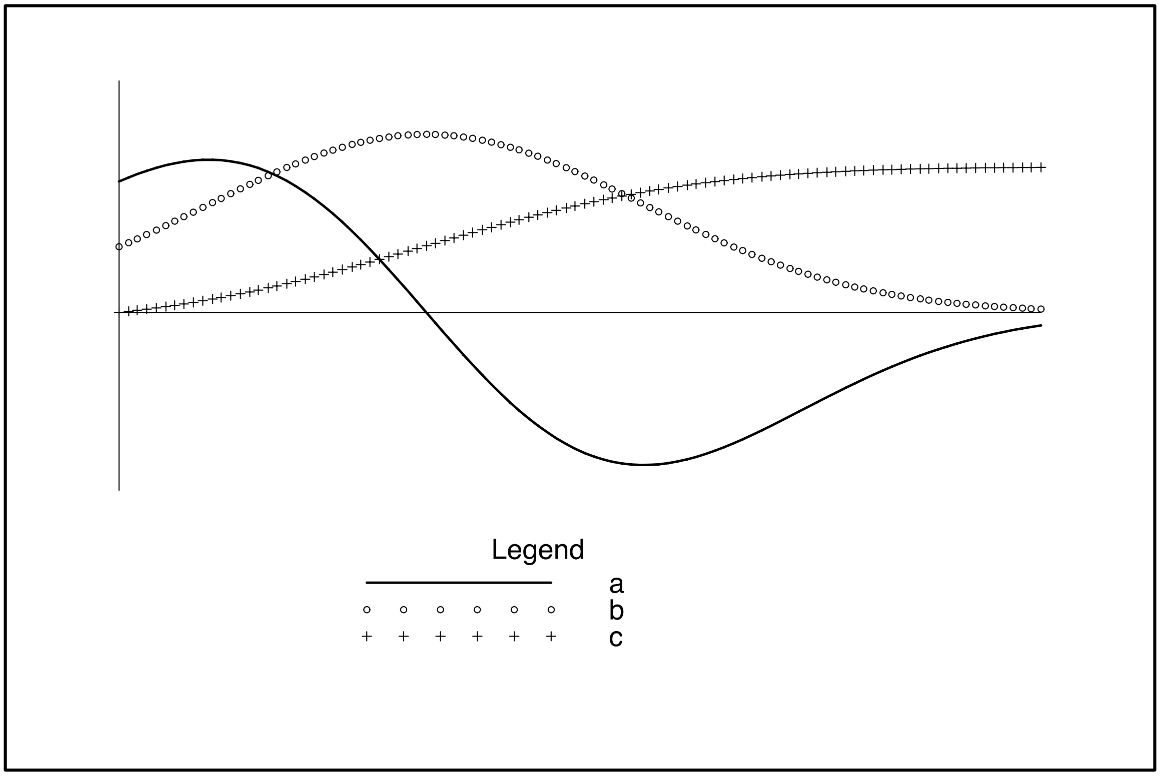
Math 118 Midterm 1 Solutions

**1.** The figure below shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.




**c is position, b is velocity, and a is acceleration.**

**2.** The time for a chemical reaction, $T$ (in minutes), is a function of the amount of catalyst present, $a$ (in milliliters), so $T = f(a)$.

Explain, in words, the meaning of the following expressions or equations. (Your explanations should be understandable by someone who hasn't taken calculus. Units may be helpful.)

(a) $f(5) = 18$
(b) $f^{-1}(10)$
(c) $f'(5) = -3$

**(a) If there are 5 mL of catalyst present, the reaction takes 18 minutes. (b) The amount (in mL) of catalyst necessary for the reaction to take 10 minutes. (c) Roughly, the reaction will take 3 fewer minutes if we increase the amount of catalyst from 5 to 6 mL.**

**3.** Find the equation of the line tangent to the graph of $f(x) = x^3$ when $x = -2$.

**The slope of the tangent line is the derivative,** $f'(-2)$**. Since** $d/dx(x^3) = 3x^2$**,** $f'(-2) = 3(-2)^2 = 12$**. The point on the graph corresponding to** $x = -2$ **is** $(-2, f(-2)) = (-2, (-2)^3) = (-2, -8)$**. Thus the tangent line is the line of slope 12 through the point (-2, -8), or the line** $y - (-8) = 12(x - (-2))$**, or** $y + 8 = 12x + 24$**, or** $y = 12x + 16$**.**

**4.** A certain pop star is paying me to advertise her new chewing gum, Britney Spearmint®, in my calculus classes. (Example: "... so we see that the derivative is positive. Something else that's positive is the feeling I get when I chew Britney Spearmint Gum®!") The amount that she pays me for a lecture is proportional to the logarithm of the number of times that I mention the gum in the lecture. If I get \$92.66 for mentioning it 10 times, how many times would I have to mention it to get \$100?

**(Note: It doesn't matter which log we use; we'll get different constants of proportionality but the same answer.) If we use** $\ln$**, then she pays** $f(x) = k\ln(x)$**. Plug in** $f(10) = 92.66$ **and solve to get** $k \approx 40.243$**. Now set** $f(x) = 40.243\ln(x) = 100$ **and solve:** $\ln(x) = 100/40.243 \approx 2.485$**, so** $x \approx e^{2.485} \approx 12$**. So I need to mention it 12 times. If we use** $\log_{10}$**, then the calculations are similar, but** $k = 92.66$**.**

**5.** Let $f(t)$ be the number of new cars and trucks sold in the United States in year $t$. We have the following data (from econstats.com and NADA).

| $t$ | $f(t)$ |
|---|---|
| 2001 | 17,121,900 |
| 2002 | 16,817,500 |
| 2003 | 16,634,700 |
| 2004 | 16,866,500 |

(a) Estimate $f'(2004)$.

(b) Use your answer to (a) to estimate the number of new cars and trucks sold in the United States in 2006.

**(a) We'll use the average rate of change from 2003 to 2004, namely (16,866,500 - 16,634,700)/(2004-2003)=231,800. (b) We want** $f(2006)$**. The number sold is increasing by roughly** $f'(2004) \approx$ **231,800 per year, so in the two years from 2004 to 2006 it will increase by roughly 2·231,800=463,600, so** $f(2006)$ **is roughly 16,866,500 + 463,600 = 17,330,100.**