Math 118 Solutions to 2nd Midterm

1. Find an equation for the tangent line to the ellipse $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

Use implicit differentiation. Take d/dx of both sides to get $2x + y + xy' + 2yy' = 0$, solve to get $y' = \frac{-2x - y}{x + 2y}$, plug in $(1, 1)$ to get $y' = -1$. An equation for the tangent line is $y - 1 = -1(x - 1)$, or $y = -x + 2$.

2. Use linear approximation to estimate $(2.001)^5$.

As always, linear approximation is $f(x) \approx f(a) + f'(a)(x - a)$. Here, use $f(x) = x^5$ and $a = 2$, so $f'(x) = 5x^4$, $f(a) = 2^5 = 32$, and $f'(a) = 5(2^4) = 80$. Thus $(2.001)^5 = f(2.001) \approx 32 + 80(.001) = 32.08$.

3. Suppose that $B(t)$ is the number of bananas that I eat on the t^{th} day of the current year, and $C(b)$ is how much it costs to buy b bananas. On January 5 of this year, I ate 20 bananas, and that number was decreasing by 3 bananas per day. The price of a banana on January 5 was \$0.50. Calculate the following quantities on January 5 of this year, and write a sentence interpreting each value.

$B(5), \frac{dB}{dt}, \frac{dC}{db}, \frac{dC}{dt}$.

(a) $B(5) = 20$: I ate 20 bananas on January 5. (b) $\frac{dB}{dt} = -3$: On January 5, the number of bananas I was eating in a day was decreasing at a rate of 3 bananas per day. (So on January 6, for example, I'd eat approximately 17 bananas.) (c) $\frac{dC}{db} = .5$: On January 5, the total amount of money I spent on bananas in a day would increase by \$0.50 for every extra banana I ate. (d) $\frac{dC}{dt} = \frac{dC}{db}\frac{dB}{dt} = (.5)(-3) = -1.5$ (or, $\frac{d}{dt}(C(B(t))) = C'(B(t))B'(t) = (.5)(-3) = -1.5$): On January 5, the total amount of money I spent on bananas in a day was decreasing by \$1.50 per day.

4. Bears have a lot of trouble finding comfortable furniture for their caves. To help them out, Claire has started her own company, Claire's Chairs for Bears' Lairs, Inc. Her fixed costs are \$5000, and each chair she manufactures costs her an additional \$10. In order to sell q chairs, she needs to set the price at \$$p$, where $p = -5q + 4000$.

(a) Express the company's costs $C(q)$ as a function of the quantity sold q.
(b) Express the company's revenue $R(q)$ as a function of the quantity sold q.
(c) Express the company's profit $\pi(q)$ as a function of the quantity sold q.
(d) How many chairs should Claire produce to earn the largest possible profit, and what is that profit?

(a) $C(q) = 5000 + 10q$. (b) $R(q) = pq = (-5q + 4000)q = -5q^2 + 4000q$. (c) $\pi(q) = R(q) - C(q) = -5q^2 + 3990q - 5000$. (d) We want to maximize $\pi(q)$. First, we need to find the endpoints of the interval of possible values of q. Clearly $q \geq 0$. Also, $p \geq 0$. Since $p = -5q + 4000$, this gives $q \geq 800$. So our interval is $[0, 800]$. To maximize, we need to find the critical points. $\pi'(q) = -10q + 3990$, which exists everywhere, so the only critical point is $q = 399$. Now we check the critical point and end points: $\pi(0) = -5000$, $\pi(399) = 791,005$, and $\pi(800) = -13,000$. So Claire should produce 399 chairs and make a profit of \$791,005.