Math 118 Practice 2nd Midterm Solutions

1. What are the maximum and minimum values of the function $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$, and where do they occur?

Check endpoints and critical points. $f'(x) = 3x^2 - 3$, **which exists everywhere, so set** $f'(x) = 0$: $3x^2 - 3 = 0$, **or** $x^2 = 1$, **or** $x = \pm 1$. **Only** $x = 1$ **is in the interval** $[0, 3]$.

x	$f(x)$
0	**1**
1	**-1**
3	**19**

So the maximum is 19, which occurs at $x = 3$, **and the minimum is -1, which occurs at** $x = 1$.

2. Use linear approximation to estimate $\ln(0.9)$.

$\mathbf{f(x) \approx f(a) + f'(a)(x - a)}$. **Here,** $a = 1$ **and** $f'(x) = 1/x$, **so** $f(a) = \ln(1) = 0$ **and** $f'(a) = 1/1 = 1$, **and we have** $\ln(x) \approx 0 + 1(x - 1) = x - 1$. **So** $\ln(0.9) \approx 0.9 - 1 = -0.1$

3. Find an equation for the line tangent to the graph of $xy + y^2 = 4$ at the point (3,1).

First, find dy/dx **by implicit differentiation. Take the derivative** d/dx **of both sides:** $x \cdot dy/dx + y + 2y \cdot dy/dx = 0$, **and solve for** $dy/dx = -y/(x + 2y)$. **So at (3,1), the slope is -1/(3+2)=-1/5, and an equation for the tangent line is y-1=-(1/5)(x-3), or y = -(1/5)x + 8/5.**

4. Given: $r(2) = 4$, $s(2) = 1$, $s(4) = 2$, $r'(2) = -1$, $s'(2) = 3$, $s'(4) = 3$. Compute the following derivatives, or state what additional information you would need to be able to do so.

(a) $H'(2)$ if $H(x) = r(x) \cdot s(x)$

(b) $H'(2)$ if $H(x) = \sqrt{r(x)}$

(c) $H'(2)$ if $H(x) = r(s(x))$

(d) $H'(2)$ if $H(x) = s(r(x))$

(a) $r(2)s'(2) + r'(2)s(2) = (4)(3) + (-1)(1) = 11$. **(b)** $(1/2)(1/\sqrt{r(2)})r'(2) = (1/2)(1/2)(-1) = -1/4$. **(c)** $r'(s(2)) \cdot s'(2) = r'(1) \cdot 3$ – **need to know** $r'(1)$. **(d)** $s'(r(2)) \cdot r'(2) = s'(4) \cdot (-1) = (3)(-1) = -3$.

5. Jungle Jane sells robotic monkeys over the internet. To sell q robomonkeys per month, she needs to set the price at $p = 125 - \frac{q}{25}$ dollars per robomonkey. If she has fixed costs of \$10,000 per month, and each robomonkey costs her \$25 to make, how many robomonkeys should she make each month to maximize her profit?

Profit = revenue - cost, or $\pi = R - Q$. **The revenue is** $R = pq = (125 - \frac{q}{25})q = 125q - \frac{q^2}{25}$, **and the cost is** $C = 10,000 + 25q$, **so the profit is** $\pi = 125q - \frac{q^2}{25} - 10,000 - 25q = 100q - \frac{q^2}{25} - 10,000$. **The endpoints are** $q = 0$ **and** $q \to \infty$. **Next, we find the critical points.** $\pi' = 100 - \frac{2q}{25}$, **which exists everywhere, so we set it equal to zero:** $100 - \frac{2q}{25} = 0$, **or** $100 = \frac{2q}{25}$, **or** $2500 = 2q$, **or** $1250 = q$. **This is the only critical point, and** $\pi'' = 2/25 > 0$, **so the maximum must occur there: she should make 1250 robomonkeys each month.**