Math 118 Practice Final Exam Solutions

1. When I first moved to Atlanta, it rained every day for two weeks. One day, I decided to measure how much it was raining. Here is a selection of the data that I gathered (t is the number of hours after noon, and $f(t)$ is the number of inches of rain that had fallen since noon).

t	3.0	3.2	3.4	3.6	3.8
$f(t)$	0.82	0.95	1.05	1.10	1.32

(a) What was the average rate at which the rain was falling between $t = 3.0$ and $t = 3.8$?
(b) Estimate $f'(3.0)$ (including units).

(a) (amt. of rain fallen)/(elapsed time) $= \dfrac{1.32 - 0.82}{3.8 - 3.0} = .625$ **inches per hour.**

(b) $f'(3.0) \approx \dfrac{f(3.2) - f(3.0)}{0.2} = \dfrac{0.13}{0.2} = .65$ **inches per hour.**

2. Compute the following derivatives:

(a) $\dfrac{d}{dx}(\cos(3x^2+1))$

(b) $\dfrac{d}{dx}\left(\dfrac{4^x}{7-x}\right)$

(c) $\dfrac{d}{dt}(5 + 3t^2e^{-t})$

(a) $-6x\sin(3x^2+1)$ **(chain rule)**

(b) $\dfrac{(7-x)(\ln 4)(4^x) + 4^x}{(7-x)^2}$ **(quotient rule)**

(c) $-3t^2e^{-t} + 6te^{-t}$ **(product rule)**

3. Compute the following integrals:

(a) $\displaystyle\int_{-\pi}^{0} \sin(t)\, dt$

(b) $\displaystyle\int \frac{2}{t}\, dt$

(c) $\displaystyle\int_{1}^{3} \frac{2}{x^3}\, dx$

(d) $\displaystyle\int_{-2}^{2} |x|\, dx$

(a) $-\cos(t)\ |_{-\pi}^{0} = -\cos 0 - (-\cos(-\pi)) = -2$

(b) $2\ln|t| + C$

(c) $-x^{-2}\ |_1^3 = -\frac{1}{9} + \frac{1}{1} = \frac{8}{9}$

(d) 4 (draw the picture, or break up into two pieces ($x \le 0$ **and** $x \ge 0$**))**

4. Use linear approximation to estimate $\ln(0.9)$. (Hint: $a = 1$ may be a good choice.)

$\mathbf{f(x) \approx f(a) + f'(a)(x - a)}$**. Here,** $a = 1$ **and** $f'(x) = 1/x$**, so** $f(a) = \ln(1) = 0$ **and** $f'(a) = 1/1 = 1$**, and we have** $\ln(x) \approx 0 + 1(x-1) = x - 1$**. So** $\ln(0.9) \approx 0.9 - 1 = -0.1$

5. (30 pts) A truck full of fashionable shoes leaves the factory at noon and travels along a straight road. The truck's velocity (in miles per hour) is given by $v(t) = 30t - 3t^2$, where t is the number of hours since noon. If you live 10 miles down the road from the factory, will the truck have passed your house by 1:00 pm?

The distance traveled after 1 hour is given by the integral $\int_0^1 v(t)\,dt = \int_0^1 (30t - 3t^2)\,dt = (15t^2 - t^3)\mid_0^1 = (15-1) - (0-0) = 14$. **The truck has traveled 14 miles after an hour, so it has passed your house.**

6. You've just been hired as the new president of Delta Airlines. Your underlings tell you that the regular air fare between Atlanta and Austin is $500. Delta flies 747s (which can hold up to 380 people) on this route, and averages 300 passengers. Market research indicates that each $1 fare reduction would attract, on average, 1 more passenger for each flight (and conversely: each $1 fare increase would reduce the average number of passengers per flight by 1). How should you set the fare to maximize Delta's revenue?

Revenue = (price p)(quantity sold q). Let x be the amount the price has increased from $500. Then $p = 500 + x$, **and** $q = 300 - x$, **so we'll maximize the revenue** $r(x) = (500 + x)(300 - x)$. **The biggest** x **could be is 300 (we can't have negative passengers), and the smallest it could be is -80 (the plane can't hold more than 380 people). Now we find the critical points:** $r'(x) = 300 - x - (500 + x)$ **(product rule), or** $r'(x) = -200 - 2x$. **The only critical point is at** $x = -100$, **but that isn't in our interval [-80, 300], so the maximum must occur at an endpoint.** $r(300) = 0$ **and** $r(-80) = 420 * 380$, **so we should choose** $x = -80$ **and set the fare at $420.**

7. Let $f(t)$ be the total number of gallons of delicious chocolate milk that Hilda has consumed by age t (years). Interpret the following in practical terms. (HINT: units!)

(a) $f(14) = 400$
(b) $f^{-1}(50) = 6$
(c) $f'(12) = 50$
(d) $(f^{-1})'(450) = 1/70$

(a) By age 14, Hilda had consumed 400 gallons of chocolate milk.

(b) Hilda had consumed 50 gallons of chocolate milk by age 6.

(c) At age 12, Hilda was consuming roughly 50 gallons of chocolate milk per year.

(d) At the time that Hilda had consumed 450 gallons of chocolate milk, it was taking her approximately one year to drink another 70 gallons (or, it was taking her about 1/70 of a year to drink another gallon).

8. I'm filling a spherical balloon with water. The volume V of the balloon depends on the radius r: $V = \frac{4}{3}\pi r^3$. After t seconds, the radius is $r(t)$ cm.

(a) Explain in words the meanings of the following derivatives (give units): dV/dr, dr/dt, dV/dt.
(b) The radius is increasing at 2 cm/sec. At what rate (cm^3/sec) is the volume increasing when the radius is 10 cm?

(a) dV/dr is the rate, in cm^3/cm, at which the volume increases as the radius increases; roughly, it's the amount the volume will increase if the radius increases by 1 cm. dr/dt is the rate, in cm/sec, at which the radius increases; roughly, it's the amount the radius will increase in 1 sec. dV/dt is the rate, in cm^3/sec, at which the volume increases; roughly, it's the amount the volume will increase in 1 sec.

(b) Chain rule! $V = \frac{4}{3}\pi r(t)^3$, **so** $dV/dt = \frac{4}{3}\pi \cdot 3r(t)^2 \cdot dr/dt = \frac{4}{3}\pi \cdot 3 \cdot 10^2 \cdot 2 = 800\pi \approx 2513$ **cm^3/sec.**