Math 118 Project #1

Work in groups of three to five people. Write a report of your results (with complete sentences, please, and graphs and figures as appropriate) – one report per group, pledged by all of the group members. Think of a classmate who hasn't thought about these questions as the target audience for your report. *You may not discuss your work with members of any other group.*

For this project we will look at two sets of data, one for U.S. population and one for U.S. yearly per capita energy consumption. If $P(t)$ is the population and $E(t)$ the per capita energy consumption, then their product $T(t) = P(t) \cdot E(t)$ is the *total* yearly energy consumption in the U.S. We will consider what these functions and their derivatives tell us, and how the derivative for $T(t)$ relates to the derivatives of $P(t)$ and $E(t)$.

Here are the data (from the Census Bureau and the Department of Energy). The population is measured mid-year, in millions; yearly per capita energy consumption is measured in million BTUs.

Year	Population $P(t)$	Per Capita Energy Use $E(t)$
1992	255.0	335
1993	257.8	337
1994	260.3	339
1995	262.8	342
1996	265.2	350
1997	267.6	348

1. (a) Estimate the rates of change of population and per capita energy use in 1997. (That is, estimate $P'(1997)$ and $E'(1997)$.) What are the units?

(b) Use these data to estimate the population in 1999. The actual population in 1999 was 272.7 million. What might explain the difference between your estimate and the real population?

(c) Now use these data to estimate the population in 2007. The actual population was 301.6 million. What might explain why your estimate for 2007 was so much worse than your estimate for 1999?

2. (a) For $t = 1993$, 1994, and 1995, compute $T(t)$ and estimate $P'(t)$, $E'(t)$, and $T'(t)$. What are the units?

(b) We might think that because T is the product of P and E, its derivative is the product of P' and E', that is, that $T'(t) = P'(t) \cdot E'(t)$. Does this seem to be true? Do the units work out? What's going on?

(c) We could also estimate the derivative of the sum $P(t) + E(t)$. Why aren't we doing that?

3. (a) Usually we think of the U.S. population $P(t)$ as a smooth function of time. To what extent is this justified? What happens if we zoom in at a point on the graph? What about events such as the Louisiana Purchase? Or the moment of a baby's birth?

(b) What do we in fact mean by the rate of change of the population at a particular time t?

(c) Give another example of a real-world function which is not smooth but is usually treated as such.