1. (10 pts) Find $\lim\limits_{x \to 0} \dfrac{\sin x^2}{x}$.

Plug in, set $\dfrac{\sin 0}{0} = \dfrac{0}{0}$ - undefined.

Use L'Hopital's Rule!

$$\lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} \frac{(\sin x^2)'}{x'} = \lim_{x \to 0} \frac{(\cos x^2) 2x}{1} = \frac{1 \cdot 2 \cdot 0}{1} = 0$$

2. (15 pts) Find the maximum and minimum values of the function $f(x) = x^3 - 3x$ on the interval $[-3, 2]$.

Find critical pts: $f'(x) = 3x^2 - 3$. Exists everywhere, so set $f' = 0$:

$$0 = 3x^2 - 3, \quad 3 = 3x^2, \quad 1 = x^2, \text{ so } x = \pm 1.$$

Endpts are $-3, 2$.

Check:

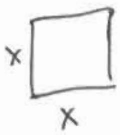
x	$f(x) = x^3 - 3x$
1	$1 - 3 = -2$
-1	$-1 + 3 = 2$
-3	$-27 + 9 = -18$
2	$8 - 6 = 2$

So the max. value is 2, & the min. value is -18

3. Let $p(t)$ be the number of squirrels living on campus t months after January 1st. What is the average number of squirrels on campus during the first six months of the year?

$$\text{ave} = \frac{\int_0^6 p(t)\,dt}{6-0}$$

4. The area A of a square is increasing at the rate of 8 in.^{2} /sec. At what rate is the edge length x increasing when each edge is 4 in.? (Give units.)



$$A = x^2$$

So $\frac{dA}{dt} = 2x\,\frac{dx}{dt}$

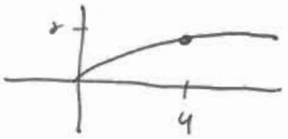
When the edge length is 4 (ie, $x=4$) & $\frac{dA}{dt} = 8$, we get

$$8 = 2\cdot 8\cdot \frac{dx}{dt}$$

$$\frac{1}{2} = \frac{dx}{dt}$$

Edge length is increasing at $\frac{1}{2}$ in/sec.

5. (a) Find the tangent line to the graph of the function $f(x) = \sqrt{x}$ at the point $(4, 2)$.



slope is $f'(4)$.

$f'(x) = (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

So $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$

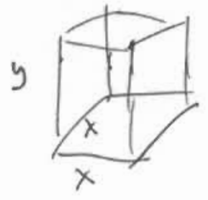
Tangent line is $y - 2 = \frac{1}{4}(x-4)$, or $y = \frac{1}{4}x + 1$

(b) Use your answer to (a) to estimate $\sqrt{3}$. (If you didn't get an answer to (a), use the line $y = 3x + 5$.)

$\sqrt{3} = f(3) \approx \frac{1}{4}\cdot 3 + 1 = 1\,3/4$

(If you used $y = 3x+5$, you should get $3\cdot 3 + 5 = 14$)

6. A rectangular box has a square base with edges at least 1 in. long. It has no top, and the total area of its five sides is 300 in.^{2} What is the maximum possible volume of such a box?



Maximize volume $= x^2 y$

$300 = \text{area} = x^2 + 4xy$, or $4xy = 300 - x^2$

$y = \dfrac{300 - x^2}{4x}$

So maximize $x^2 y = x^2\left(\dfrac{300 - x^2}{4x}\right) = \dfrac{300x - x^3}{4}$

endpts: $x \geq 1$ (given)

y must be positive, so $\dfrac{300 - x^2}{4x} > 0$, or $300 > x^2$, or $\sqrt{300} > x$ (since $x > 0$).

So the interval is $[1, \sqrt{300})$

Find critical pts: $\left(\dfrac{300x - x^3}{4}\right)' = \dfrac{300 - 3x^2}{4}$

This always exists, so set $= 0$.

$\dfrac{300 - 3x^2}{4} = 0$, $300 - 3x^2 = 0$, $300 = 3x^2$,

$100 = x^2$,

$+10 = x$

x	volume
1	$\frac{299}{4}$
10	$\frac{2000}{4} = 500$
$\to \sqrt{300}$	$\to 0$

So the max. vol. is 500 in^3.

EXTRA CREDIT What is the maximum possible volume of a sphere?