Math 119 Midterm 1 Solutions

1. On the moon the acceleration due to gravity is 5 ft/sec^2. A moon-cantaloupe is dropped from the top of a moon-tower and hits the moon-ground in 20 seconds. How tall is the tower? (Give your answer in feet.)

Velocity $v(t) = \int -5\,dt = -5t + C$. Because it's dropped, the initial velocity is 0, so $v(t) = -5t$. Height $h(t) = \int -5t\,dt = -\frac{5}{2}t^2 + C$, where C is the unknown height of the tower. We know that $h(20) = 0$, so $-\frac{5}{2} \cdot 20^2 + C = 0$, or $C = 1000$.

2. Explain whether the following improper integrals converge or diverge. (You don't necessarily have to compute the integral.)

(a) $\int_1^\infty \sin^2 x\,dx$

Diverges - the integrand oscillates.

(b) $\int_1^\infty \frac{1}{x^{10}}\,dx$

Converges. (Compute it, or recall that $\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$.

(c) $\int_1^\infty \frac{\sin^2 x}{x^{10}}\,dx$

Converges, by comparison with (b). ($0 \le \sin^2 x \le 1$.)

3. Compute $\frac{d}{dt}\int_1^{\sin t} \cos(x^2)\,dx$.

(This is problem 6.4.25 from your homework.) Use the FT of C and the chain rule. We want $\frac{d}{dt}\int_1^u \cos(x^2)\,dx$, where $u = \sin t$. $\frac{d}{dt}\int_1^u \cos(x^2)\,dx = \cos(u^2) \cdot \frac{du}{dt} = \cos(\sin^2 t) \cdot \cos t$.

4. The graph of $f(x)$ is given below. Let $F'(x) = f(x)$. **Please don't get F and f mixed up while doing this problem!**

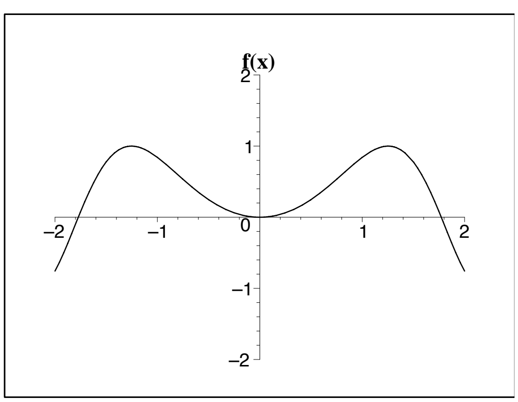
(a) What are the critical points of $F(x)$?

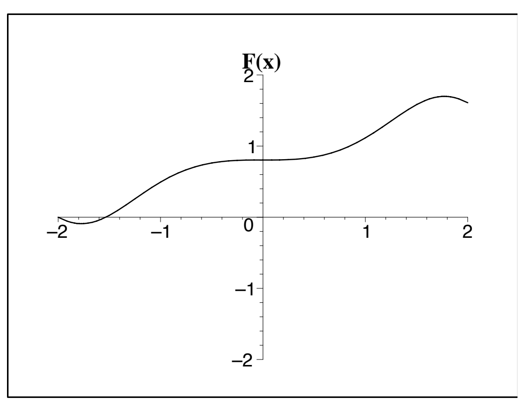
Critical points are where the derivative $F'(x) = f(x)$ is zero, namely at -1.8, 0, and 1.8.

(b) Which critical points are local maxima, which are local minima, and which are neither?

The derivative $f(x)$ goes from negative to positive at -1.8, so that's a local minimum. It goes from positive to negative at 1.8, so that's a local maximum. It goes from positive to positive at 0, so that's neither.

(c) Sketch the graph of $F(x)$ on the bottom axes, given that $F(-2) = 0$.

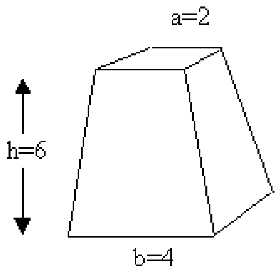




5. It's the year 2031, and Agnes Scott is erecting a statue of you, in honor of your efforts to end the war between North and South Dakota. Your statue will sit on a block of granite in the shape of a truncated pyramid, also called a frustum. It has a square base with side length $b = 4$ meters, height $h = 6$ meters, and a square top with side length $a = 2$ meters (see the figure below).

(a) Set up an integral that gives the volume of the frustum. (HINT: What is the area of a horizontal cross section?)

(b) Evaluate the integral to find the volume.



(a) If we included the top of the pyramid, the height would be 12 meters. So, using similar triangles, a horizontal cross section h meters above the ground is a square of side length $4 - \frac{h}{3}$. Thus the volume of a thin (height $= \Delta h$) horizontal slice is roughly $\left(4 - \frac{h}{3}\right)^2 \Delta h$, and the integral giving the volume is $\int_0^6 (4 - \frac{h}{3})^2 \, dh$.

(b) 56