Math 119 Midterm #2 Solutions

1. The region bounded by the curves $y = e^x$, $y = 0$, $x = -1$, and $x = 1$ is rotated about the x-axis. Compute the volume of the resulting solid.

(This is problem 8.2.5 from your homework.) The volume of each vertical slice of width dx **is** $\pi y^2 dx = \pi(e^x)^2 dx = \pi e^{2x} dx$**. The total volume is the sum (integral) of all the slices, so it's** $\int_{-1}^{1} \pi e^{2x}\,dx = \frac{\pi}{2} e^{2x}|_{-1}^{1} = \frac{\pi}{2}(e^2 - e^{-2})$**.**

2. A monkey, a lion, and a robot have one donut to share. They divide it as follows. First they divide it into fourths, each taking a quarter. Then they divide the leftover quarter into fourths, each taking a quarter, and so on. How much of the donut does the monkey end up with?

The monkey gets a quarter, plus a quarter of a quarter, plus a quarter of a quarter of a quarter, plus. . . , or $1/4 + (1/4)^2 + (1/4)^3 + (1/4)^4 + \cdots = \frac{1}{4} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$**.**

3. Use a second-degree Taylor polynomial to estimate $\cos(-0.1)$.

We'll use a Taylor polynomial centered at 0. $P_2(x) = \cos 0 + (\cos' 0)x + (\cos'' 0)x^2/2 = 1 + 0x - 1/2x^2$**. So** $\cos(-0.1) \approx P_2(-0.1) = 1 - (-0.1)^2/2 = 0.995$**.**

4. Determine whether the following series converge or diverge. Be sure to give reasons!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

(a) diverges (limit comparison to $\sum \frac{1}{n}$**) (b) converges (alternating series test) (c) converges (ratio test) (d) converges (ratio test, or limit comparison to the convergent geometric** $\sum \frac{2^n}{3^n}$**, or notice that it's the sum of the convergent geometric** $\sum \frac{1}{3^n}$ **and** $\sum \frac{26n}{3^n}$**)**