Math 119 Practice Final

The actual final will of course be shorter than this. You may use a calculator, but **not any of its calculus functions.** You don't need to memorize the formulas for the Fourier coefficients; I'll give them to you if you need them.

1. Chapter 6 Review (p. 306), #63.
2. Chapter 7 Review (p. 361), #1-146 as needed.
3. Chapter 9 Review (p. 471), #23-41 odd.
4. Chapter 11 Review (p. 595), #3, 5, 7, 25, 27.
5. Use a second-degree Taylor polynomial to approximate $\ln 1.3$. How big might the difference between your approximation and the real answer be?
6. Suppose that the government pumps an extra \$1 billion into the economy. Assume that each business and individual saves 25% of its income and spends the rest, so of the initial \$1 billion, 75% is respent by individuals and businesses. Of that amount, 75% is spent, and so on. What is the total increase in spending due to the government action? (This is called the *multiplier effect* in economics.)
7. Find the length of the curve $y = x^{3/2}$ from $x = 1$ to $x = 4$.
8. Let R be the region bounded by the curve $y = x - x^2$ and the x-axis.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving the region R about the x-axis.
9. Let $z = 2 - 2i$.
   (a) Write z in the form $e^{i\theta}$.
   (b) Find $\bar{z}$.
   (c) Find z^2.
   (d) Find $|z|$.
   (e) Find $1/z$.
10. Evaluate (a) $\displaystyle\int_1^3 \frac{dx}{(x-1)^{1/3}}$ and (b) $\displaystyle\int_9^\infty \frac{x}{\sqrt{1+x^2}}\,dx$, or show that they do not exist.
11. The lifetime of a cellphone, measured in months, can be modeled by a random variable X with an *exponential distribution*, which has probability density function $f(t) = 0.01e^{-0.01t}$ $(t \geq 0)$. What is the probability that a randomly selected cellphone lasts for at least 20 months?
12. A piece of wire, stretching from $x = 0$ to $x = 10$, has density $\delta(x) = 3x^2$. Find (a) the mass of the wire, and (b) the center of mass of the wire.
13. If q Dancin' Scotty dolls are to be sold, the price must be set at $p = D(q) = 3e^{\frac{-q}{1{,}000{,}000}}$ dollars.
   (a) If 18,000 are sold, what's the price?
   (b) If the price is \$1, how many are sold?
   (c) What's the consumer surplus, if the equilibrium price is \$1?
14. Calculate $\displaystyle\frac{d}{dx}\int_5^{e^x} \cos(t^3)\,dt$.
15. We want to study the populations of whales in the ocean and of mold on the bread in my refrigerator. We know that whales live in the vast, vast ocean and have plenty of room and lots of delicious plankton to eat. At the same time, though, they're so spread out that if there aren't very many of them, they won't be able to find each other in order to breed.

We also know that mold reproduces asexually (i.e., a mold cell doesn't need another mold cell to reproduce). On the other hand, once the mold covers the bread, there'll be no place else for it to live.

(a) Differential equations modeling the two populations $P(t)$ and $Q(t)$ are given below. Which is which? That is, is $P(t)$ the whale population at time t and $Q(t)$ the mold population, or is it the other way around? Explain your reasoning. (k_1 and k_2 are positive constants.)

$$\frac{dP}{dt} = k_1 P(P - 10,000) \qquad \frac{dQ}{dt} = k_2 Q(10,000 - Q)$$

(b) What are the possible long-term behaviors of solutions to the first equation ($\frac{dP}{dt} = k_1 P(P - 10,000)$)?