Math 119 Practice Second Midterm

The actual midterm will of course be shorter than this. You may use a calculator, but **not any of its calculus functions.**

1. Section 8.6 (p. 413): #13. Section 8.8 (p. 426): #7. Chapter 8 Review (p. 428): #17, 23, 37, 55.

2. Sarah Series would like to set up a scholarship for Math 119 students, to be awarded each year, starting in one year (sorry). How much money would she have to deposit in an account which earns 7% interest a year (compounded continuously; that is, if she deposits $\$D$, then after t years she'll have $\$De^{0.07t}$) in order to award a \$1,000 scholarship each year? (Hint: As is usually the case with word problems, it helps to break the problem into small pieces. First compute how much Sarah would need to deposit now in order to award a \$1,000 scholarship next year. Then add in the amount that Sarah would need to deposit in order to award a \$1,000 scholarship in two years. And so on. . .)

3. Determine if the following series converge or diverge. Be sure to give reasons!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4 - 6n^3 + 13n}{n^5 + n^2 + 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^{n-1} + 1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(n!)(n!)}{(2n)!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{2}{n}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} a_n$, if the nth partial sum of this series is given by $s_n = \dfrac{n-1}{2n+1}$.

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n} a_n$, if you know that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a positive series that converges.

4. Find positive numbers A and B such that $0 < A \leq \displaystyle\sum_{n=0}^{\infty} \frac{1}{5^n + n^3} \leq B$.

5. Use a third-degree Taylor polynomial to estimate $\sqrt{1.1}$.

6. Compute $1/e$ to within $1/10$ of its actual value. Be sure to explain how you can be sure of the accuracy of your estimate. (Hint: $1/e = e^{-1}$, and $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$)

7. Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{4^n}$.

8. A rod of length 3 meters and density $\delta(x) = 2 + \cos x$ grams/meter is positioned along the x-axis with its left end at the origin.

(a) Where is the rod most dense?
(b) Where is the rod least dense?
(c) Is the center of mass of this rod closer to the origin, or closer to $x = 3$?
(d) What is the total mass of the rod?

(e) Where is the center of mass of the rod?