Math 321 Algebra Practice Midterm #2

1. COMPUTATIONAL PROBLEMS

(a) Find the remainder of 7^{206} when it is divided by 15.

(b) Let $f(x) = x^3 - x^2 - x - 1$.

(i) Is $f(x)$ irreducible over $\mathbb{Z}$? Why or why not?

(ii) Is $f(x)$ irreducible over $\mathbb{Q}$? Why or why not?

(iii) Is $f(x)$ irreducible over $\mathbb{Z}_2$? Why or why not?

(iv) Is $f(x)$ irreducible over $\mathbb{Z}_3$? Why or why not?

(c) How many distinguishable necklaces (with no clasp) can be made using three beads, each of which can be mauve, chartreuse, vermilion, or taupe?

2. Let R be a commutative ring and N an ideal of R. Show that the set $\sqrt{N}$ of all $a \in R$ such that $a^n \in N$ for some $n \in \mathbb{Z}^+$ is an ideal of R. ($\sqrt{N}$ is called the *radical of* N.)

3. Show that the ring $\mathbb{R}[x]/\langle x^2 \rangle$ is not an integral domain. Find three different zero divisors.

4. (a) Give an example of a field.

(b) Give an example of an integral domain that is not a field.

(c) Give an example of a commutative ring that is not an integral domain.

(d) Give an example of a ring that is not commutative.