(1) Find the maximum and minimum values of the function $f(x, y) = x^2 + 2y^2$ on the unit disk $x^2 + y^2 \leq 1$.

(2) Let $\mathbf{u}$ and $\mathbf{v}$ be two 3-dimensional vectors lying in the xy-plane, with an angle of 45° between them. If $||\mathbf{u}|| = 3$ and $||\mathbf{v}|| = 2$, find the following:
 (a) $\mathbf{u} \cdot \mathbf{v}$
 (b) $\mathbf{u} \times \mathbf{v}$

(3) Verify Gauss's (Divergence) Theorem for the solid unit ball B ($x^2 + y^2 + z^2 \leq 1$) and the vector field $\mathbf{F}(x, y, z) = (0, -z, y)$.

(4) Compute $\displaystyle\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\, dx\, dy$.

(5) State whether each of the following statements is true or false. For each that is false, give an example showing that it's false.
 (a) For every C^2 function f, $\nabla \times (\nabla f) = \mathbf{0}$.
 (b) For every C^2 function f, $\nabla \cdot (\nabla f) = \mathbf{0}$.
 (c) For every C^2 vector field $\mathbf{F}$, $\nabla \cdot (\nabla \times \mathbf{F}) = \mathbf{0}$.

(6) Use Stokes' Theorem to compute $\displaystyle\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 10$ and $\mathbf{F}$ is the vector field $\mathbf{F}(x, y, z) = (\cos(\ln(xz)),\ (x^2 + y^2)^{\frac{5}{2}},\ e^{z^2 - x})$.

(7) Give an example of a vector field that is *n*ot a gradient vector field, and explain how you can be sure that it isn't.

(8) Let S be the surface parametrized by

$$x = 1 + \cos u, \quad y = 4 + 3\sin u, \quad z = v, \quad 0 \leq u \leq 2\pi, \quad -2 \leq v \leq 2.$$

Find an equation for the tangent plane to the surface S at the point corresponding to $u = 0,\ v = 1$.

(9) Let $\mathbf{F}(x, y, z) = \mathbf{k}$. Define the following surfaces in $\mathbb{R}^3$:
 - R_1 is the unit disk in the xy-plane, oriented in the negative z direction.
 - R_2 is the upper unit hemisphere, i.e., $R_2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1,\ z \geq 0\}$, oriented in the positive z direction.
 - R_3 is the unit disk in the yz-plane, oriented in the positive x direction.

Place the following flux integrals in order from least to greatest:

$$\iint_{R_1} \mathbf{F} \cdot d\mathbf{S}, \quad \iint_{R_2} \mathbf{F} \cdot d\mathbf{S}, \quad \iint_{R_3} \mathbf{F} \cdot d\mathbf{S}$$

(10) Let D be the elliptical disk $25x^2 + 4y^2 \leq 100$. Compute the area of D in two different ways:
 (a) Using the fact that the change of variables $T(u, v) = (2u, 5v)$ maps the unit disk D^* in the (u, v)-plane one-to-one onto D.
 (b) Using Green's Theorem.

(11) Define the functions

$$g(x, y) = (g_1(x, y), g_2(x, y)) = (x^2 + 1, y^2),$$
$$f(u, v) = (f_1(u, v), f_2(u, v), f_3(u, v)) = (u + v, u, v^2), \text{ and}$$
$$h(x, y) = (h_1(x, y), h_2(x, y), h_3(x, y)) = f(g(x, y)).$$

 (a) Compute $\mathbf{D}h(1, 1)$.
 (b) What is $\dfrac{\partial h_3}{\partial y}(1, 1)$?

(12) Let R be the region in $\mathbb{R}^3$ bounded by the sphere of radius 3, the sphere of radius 4, the cone $z = -\sqrt{x^2 + y^2}$, and the cone $z = -2\sqrt{x^2 + y^2}$. Which of the following integrals represents the volume of R?

(a) $\displaystyle\int_3^4 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{x^2+y^2}}^{-2\sqrt{x^2+y^2}} dz\, dy\, dx$

(b) $\displaystyle\int_3^4 \int_0^{2\pi} \int_{\frac{3\pi}{4}}^{\frac{5\pi}{6}} d\phi\, d\theta\, d\rho$

(c) $\displaystyle\int_3^4 \int_0^{2\pi} \int_{-2r^2}^{-r^2} r\ dz\, dr\, d\theta$

(d) None of the above.

EXTRA CREDIT Explain in what sense Stokes' Theorem is a version of the Fundamental Theorem of Calculus.