Name: mean: 82.2 Median: 83 (2 hr exam)
(good distribution)

Math 6C - Calculus IIC
Midterm Exam: November 23, 2004

Please be sure to **show all of your work** and clearly label your answers.

1. (16pts) Please give the equation of the plane containing the points $(5,2,3)$, $(4,1,2)$, and $(6,10,-1)$.

4 $\vec{v}_1 = \langle 1, 1, 1 \rangle$

$\vec{v}_2 = \langle 1, 8, -4 \rangle$

6 $\vec{n} = \vec{v}_1 \times \vec{v}_2 = \langle -12, 5, 7 \rangle$

6 (50) plane:

4 $\langle -12, 5, 7 \rangle \cdot \langle x-5, y-2, z-3 \rangle = 0$

2 $-12x + 60 + 5y - 10 + 7z - 21 = 0$

$-12x + 5y + 7z = -29$

2. (12pts) Please give a vector pointing in the direction of the line of intersection of the two planes $3x + 4y = 5$ and $x + 2y + z = 2$.

$\bar{n}_1 = \langle 3, 4, 0 \rangle$

$\bar{n}_2 = \langle 1, 2, 1 \rangle$

$\bar{n}_1 \times \bar{n}_2 = \langle 4, -3, 2 \rangle$

3. (16pts) Suppose that $f(x,y) = y^2 e^{x^2 y}$. Use the differential of f to estimate $f(0.02, 2.95)$.

2pts. $(a,b) = (0,3)$

6 pts.
$$f_x = 2xy^3 e^{x^2 y} \qquad f_x(0,3) = 0$$
$$f_y = 2y e^{x^2 y} + x^2 y^2 e^{x^2 y} \qquad f_y(0,3) = 6$$
2pts.

1pt $f(0,3) = 9$

4pts $f(0.02, 2.95) \approx f(0,3) + f_x(0,3)\Delta x + f_y(0,3)\Delta y$

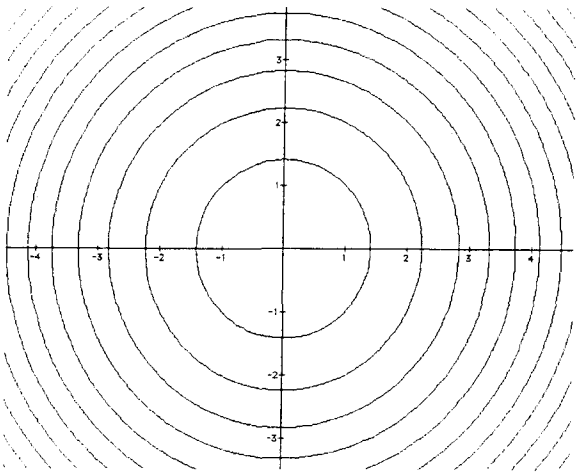
$= 9 + 0(0.02) + 6(-0.05)$

1 pts. $= 9 - 0.3$

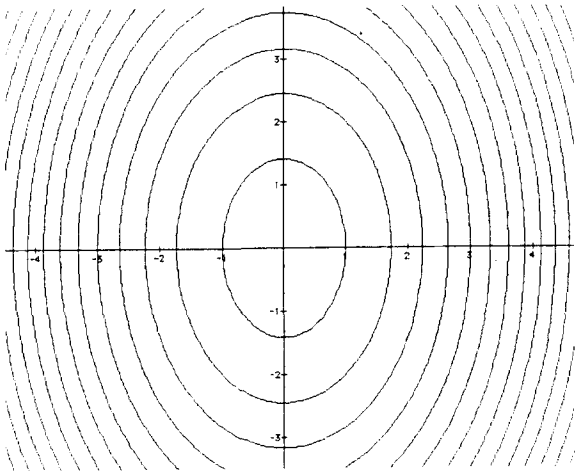
$= \boxed{8.7}$

4. (12pts) Which of the following is a contour plot of the function $f(x, y) = (x + y)^2$. **Please explain your reasoning.**

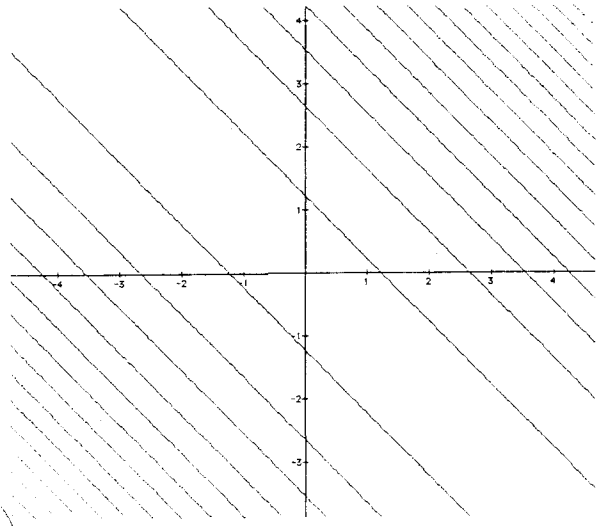
I



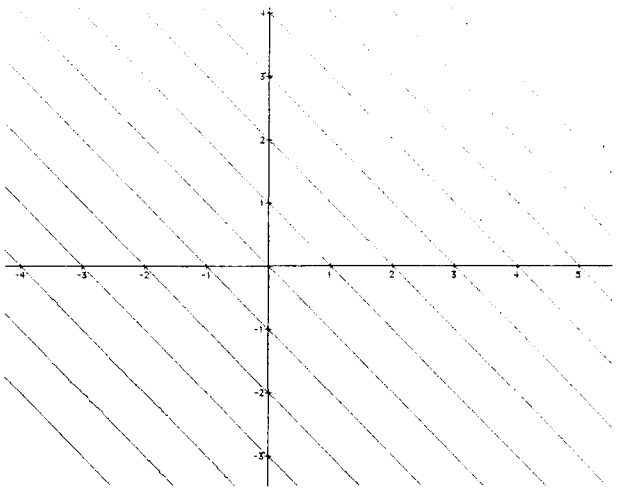
II



III



IV



$f(x,y) = (x+y)^2 = c \Rightarrow$ $x+y = \sqrt{c}$.

4 lines, slope $= -1$. As c changes evenly, the y-intercepts do not change evenly.

5. (16pts) The surface of a nearby lake can by represented region in the xy-plane such that the lake's depth (in meters) at the point (x, y) is given by $f(x, y) = 400 - x^2y^2$. Donald Trump is in the lake at position $(1, -2)$. If Mr. Trump wants to swim back to his boat, located at $(0, 0)$, what is the rate of change of the depth of the water?

6 $\left[\right.$ $f_x = -2xy^2$ $\quad f_x(1,-2) = -8$

$f_y = -2x^2y$ $\quad f_y(1,-2) = 4$

6 $\left[\right.$ $\vec{u} = \langle -1, 2 \rangle$ $\quad \vec{u} = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

4 $\left[\right.$ $D_{\vec{u}} f = -8\left(-\frac{1}{\sqrt{5}}\right) + 4\left(\frac{2}{\sqrt{5}}\right) = \frac{16}{\sqrt{5}}$

6. (28pts) True or false? For each, **please explain your reasoning**

(a) Two contours of $f(x, y)$ with different heights never intersect.

Answer

True

(b) There is exactly one linear function $f(x, y)$ whose $f = 0$ contour is $y = 2x + 1$.

False.

(c) If $\bar{v}$ and $\bar{w}$ are any vectors, then $\|\bar{v}\| + \|\bar{w}\| = \|\bar{v} + \bar{w}\|$.

False.

(d) The function $f(r, s) = rse^s$ is decreasing in the s-direction at the point $(r, s) = (-1, 2)$.

$$\frac{\partial f}{\partial s} = re^s + rse^s$$

$$\frac{\partial f}{\partial s}(-1, 2) = -e^2 - 2e^2 = -3e^2 < 0$$

So decreasing.

True.