(1) Solve the following differential equations. (Give the general solution if no initial condition is specified.)
  (a) $(t\cos(ty) + 2y)y' + y\cos(ty) + 1 = 0$
  (b) $y' = ty$
  (c) $y' + 2ty + t = 0$, $y(0) = 2$
(2) Find the general solution to the ODE $y'' + 3y' + 2y = \cos t$.
(3) Consider the initial value problem $y'' = 4y$, $y(0) = y_0$, $y'(0) = y_0'$. For what initial conditions (i.e., what values of y_0 and y_0') will the solution tend to 0 as $t \to \infty$?
(4) For each of the following ODEs, give its order, state whether it's autonomous, and state whether it's linear. If it's linear, state whether it's homogeneous or inhomogeneous.
  (a) $yy''' - t^2 y' = t \sin t$
  (b) $y'' - ty = e^t y'$
  (c) $y' - y = 0$
(5) The figure below shows some solution curves for the ODE $2(y + 2)y' = 3t^2 + 4t + 2$. Which solutions, if any, fail to exist for all time t? Are there any initial conditions for which the solution is not unique?
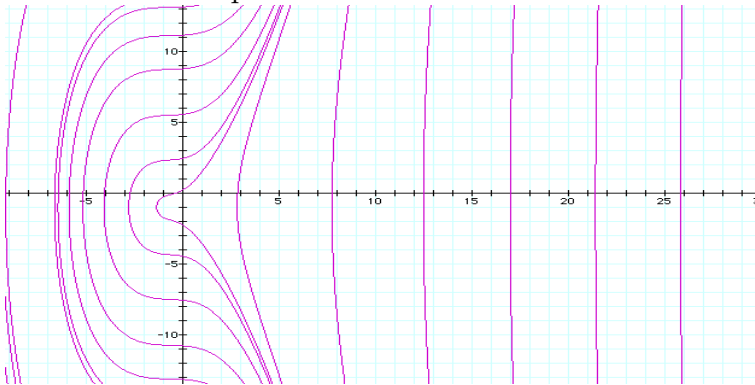
(6) Find the equilibria for the ODE $y' = y^2 - y - 2$. Are they stable, unstable, or semistable? What is the long-term behavior of the solutions?
(7) Show that $\{t, t\ln(t)\}$ is a fundamental set of solutions to the ODE $y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0$ on the interval $(0, \infty)$. Find the solution satisfying $y(1) = -1$ and $y'(1) = 2$.
(8) Consider the initial value problem $ty' = (1 + t)y$, $y(0) = 0$. How many solutions are there to this initial value problem? Does the existence and uniqueness theorem for first order differential equations apply?
(9) Suppose that sodium pentobarbitol is used to anesthetize a dog: the dog is anesthetized when its blood contains at least 45 milligrams of sodium pentobarbitol per kilogram of the dog's body weight. Suppose also that sodium pentobarbitol is eliminated exponentially from the dog's bloodstream, with a half-life of 5 hours. What single dose should be administered in order to anesthetize a 50-kg dog for 1 hour?
(10) Discuss the differences between linear and nonlinear ODEs.