Math 5 December 18, 2001 7–10 pm Name: ______________________

Your professor's name: ______________________

FINAL EXAMINATION

This exam is 11 pages long; check that you have all the pages. Show your work. Correct answers with no justification may receive little or no credit. Write complete sentences. Use correct notation and terminology. No calculators are allowed. No uncalled-for simplification is required. Use the backs of pages if you run out of space, and make sure that we can find your answers.

1.	/20
2.	/20
3.	/20
4.	/10
5.	/10
6.	/20
7.	/15
8.	/15
9.	/10
10.	/20
11.	/10
12.	/15
13.	/15
Total	/200

1. Find the derivatives of the following functions. (20 points)

   (a) $y = 3^x$

   (b) $y = 3x^5 \sin(4x)$

   (c) $y = \dfrac{3x+4}{x^3+x+2}$

   (d) $y = \left(\ln(3x^2+4)\right)^8$

2. Evaluate the following integrals. (20 points)

(a) $\int \left(x^{\frac{3}{2}} + \sin x\right) dx$

(b) $\int_{-2}^{1} |x| \, dx$

(c) $\int \frac{1}{x} \, dx$

(d) $\int_{\ln 3}^{\ln 6} 8e^{t} \, dt$

3. Below is a table of some values for the function $f(x) = x^x$. (20 points)

x	2	2.1	2.2	2.3	2.4	2.5
x^x	4	4.7	5.7	6.8	8.2	9.9

(a) Use the values in the table to compute an estimate for $f'(2.5)$.

(b) Use the values in the table to compute a left hand sum to estimate $\int_2^{2.5} x^x \, dx$.

(c) Use the values in the table to compute a right hand sum to estimate $\int_2^{2.5} x^x \, dx$.

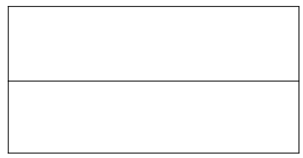
(d) How small should Δx be to guarantee that the left and right hand sums for $\int_2^{2.5} x^x \, dx$ differ by at most 0.01?

4. Suppose that at a price of \$$p$ per pound, q pounds of delicious cheese are sold. Let $q = f(p)$. If $f(10) = 300,000$ and $f'(10) = -40,000$, roughly how many pounds of delicious cheese would you expect to be sold if the price p is \$9.50? (10 points)

5. Find an equation for the tangent line to the curve $x^2 + 3y^2 = 4$ at the point $(1, 1)$. (10 points)

6. Do **ONE** of the following two problems (either A or B). If you try both, be sure to say which one you want us to grade. (20 points)

   (A) A rectangular plot of land of area 600 ft^2 is to be surrounded by a stone wall and then divided into two equal parts by a fence parallel to one side. If it costs \$10 per foot to build the stone wall and \$2 per foot to build the fence, what should the dimensions of the plot be in order for the total cost to be as small as possible?



   (B) The manager of a 100-unit apartment building knows that all units will be occupied if the rent is \$400 per month, and that one additional unit will remain vacant for each \$5 increase in rent. What rent should the manager charge to maximize revenue?

7. Find all of the critical points of the function $f(x) = x^{\frac{2}{3}} - \frac{1}{3}x^2$. (15 points)

8. Consider a ladder 26 ft long which leans against a vertical wall. At a particular instant, the foot of the ladder is 10 ft from the base of the wall and is being drawn away from the wall at a rate of 4 ft/s. How fast is the top of the ladder moving down the wall at this instant?
Useful fact: $(10)^2 + (24)^2 = (26)^2$. (15 points)

9. Suppose that at time t (where t is measured in years), the population of squirrels on the Swarthmore College campus is increasing at a rate of $6(1.03)^t$ squirrels per year. Which of the following expressions represents the total increase in the squirrel population between the fourth and tenth years? (10 points)

   (a) $6\big((1.03)^{10} - (1.03)^4\big)$

   (b) $6(\ln 1.03)\big((1.03)^{10} - (1.03)^4\big)$

   (c) $\displaystyle\int_4^{10} 6(1.03)^t dt$

   (d) $\displaystyle\int 6(1.03)^t dt$

   (e) $\displaystyle\frac{1}{6}\int_4^{10} 6(1.03)^t dt$

   (f) none of the above

10. Flea F. Hutton jumps straight up off the ground. Her initial upward speed is 96 ft/sec. (20 points)

   (a) Find formulas for her upward velocity $v(t)$ and height above the ground $s(t)$ at time t. (Recall that the acceleration due to gravity is -32 ft/sec^2.)

   (b) How high does she get, and when does she reach the highest point in her leap?

   (c) How long is she in the air?

11. Let $v(t)$ denote the velocity of a particle at time t. Correctly label each of the mathematical expressions below with one of the following phrases:

"displacement from time 4 to time 10"
"average velocity from 4 to 10"
"acceleration at time 4"
"v(x)"
"v(10) - v(4)"
"none of the above"

Please write out the correct phrase next to each expression. Do not use arrows and do not number the phrases and refer to them by number. (10 points)

Mathematical expressions:

$$\int_4^x v'(t)dt$$

$$v'(4)$$

$$\frac{v(4)+v(10)}{2}$$

$$\int_4^{10} v(t)dt$$

$$\int_4^{10} v'(t)dt$$

$$\frac{1}{6}\int_4^{10} v(t)dt$$

$$\frac{d}{dt}\int_4^{10} v(t)dt$$

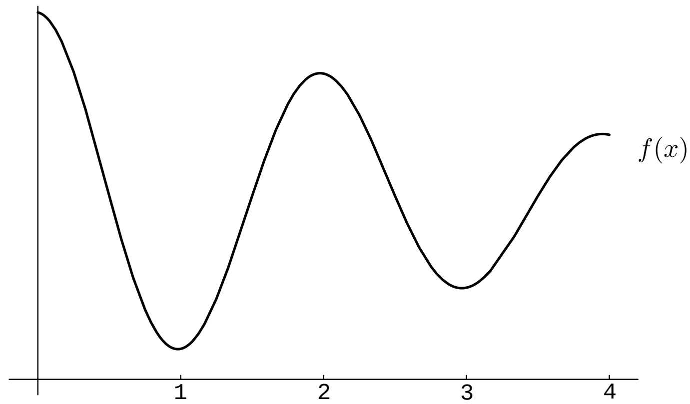
$$\frac{d}{dt}\int_4^{x} v(t)dt$$

$$\lim_{h\to 0}\frac{v(4+h)-v(4)}{h}$$

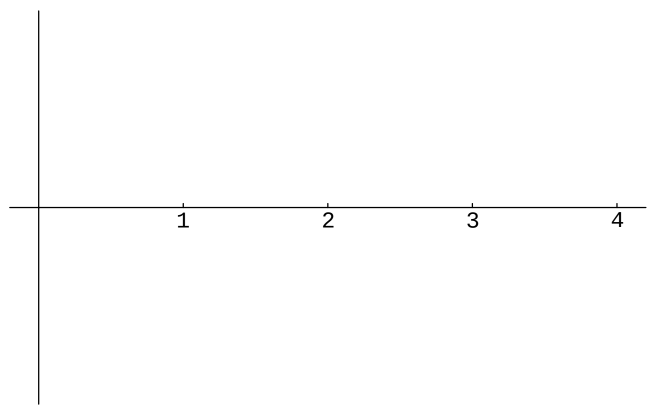
$$\lim_{h\to 0}\frac{v(4+h)+v(4)}{h}$$

$$\lim_{h\to 0}\frac{v'(4+h)-v'(4)}{h}$$

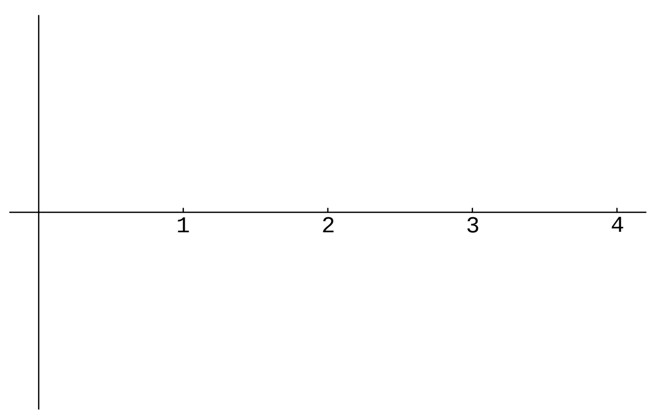
12. Consider the graph of the function $f(x)$ shown below. (15 points)



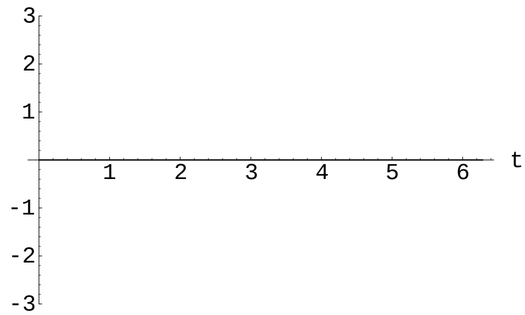
(a) Sketch the graph of a function F such that $F' = f$ and $F(0) = 0$. Where there is enough information to do so, make sure that your sketch reflects accurately where F is positive or negative, increasing or decreasing, and concave up or concave down.



(b) Sketch the graph of f', paying attention, where possible, to the same sort of details as in part (a). In addition, can you determine whether the integral of f' from $x = 0$ to $x = 4$ is positive, negative, or zero? If so, which is it? Or is more information needed before you can tell for sure? Explain clearly.



13. Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the function graphed below. (15 points)



(a) Evaluate each of the following:

(i) $g(0)$

(ii) $g(1)$

(iii) $g(2)$

(iv) $g(3)$

(v) $g(6)$

(b) On what interval(s) is g increasing?

(c) At what x value does g have a local maximum?

(d) Sketch a graph of g. Be sure to label the scale on the y-axis.