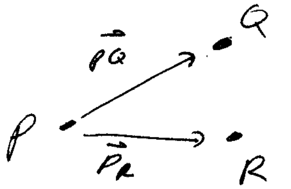
(1) (20 pts) Find an equation of the plane that passes through the points $P = (1,1,1)$, $Q = (1,2,1)$, and $R = (2,2,2)$.



Eqn. is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$, where (a,b,c) is a normal vector & (x_0,y_0,z_0) is a pt. in the plane. We already have 3 pts., so we just need to find a normal vector $\vec{n}$.

One answer: $\vec{n} = \vec{PQ} \times \vec{PR}$ will be $\perp$ to $\vec{PQ}$ & $\vec{PR}$, so $\vec{n}$ is a normal vector.

$$
\begin{aligned}
\vec{PQ} \times \vec{PR} = (0,1,0) \times (1,1,1) &= \vec{j} \times (\vec{i}+\vec{j}+\vec{k}) \\
&= \vec{j}\times\vec{i} + \vec{j}\times\vec{j} + \vec{j}\times\vec{k} \\
&= -\vec{k} + \vec{0} + \vec{i} \\
&= (1,0,-1).
\end{aligned}
$$

So an eqn for the plane (taking the pt. (x_0,y_0,z_0) to be $P=(1,1,1)$)

is $\quad 1(x-1)+0(y-1)-1(z-1)=0$

or $\quad x-1-z+1=0$

or $\quad x-z=0.$

(2) (20 pts) Consider the function $z = f(x, y) = e^y \cos x$.

(a) Find an equation of the tangent plane to the graph at the point $P = (0, 0, 1)$.

tangent plane: $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$.
at (a,b)

Here, $a=b=0$, $f(0,0) = e^0 \cos 0 = 1$, $f_x = -e^y \sin x$, so $f_x(0,0) = 0$,
and $f_y = e^y \cos x$, so $f_y(0,0) = 1$. So the tangent plane is

$$z = 1 + 0(x-0) + 1(y-0)$$

or $z = 1 + y$

(b) Use linear approximation to estimate $f(0.1, -0.2)$.

$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$ (ie, the graph looks roughly like the tangent plane). ~~the~~ Here, we have

$$f(0.1, -0.2) \approx 1 + (-0.2) = .8$$

(3) The number of hours z that it takes to cook a turkey depends on both the weight w (in pounds) of the turkey and the temperature T (in degrees F) of the oven, i.e., $z = f(w, T)$.

(a) (6 pts) Explain the meaning of the statement $f(10, 325) = 4$.

It takes 4 hours to cook a 10 lb. turkey at 325 °F.

(b) (7 pts) Explain the physical meaning of the partial derivative $\partial z/\partial w$. What are the units? Do you expect it to be positive or negative? Why?

Units are $\frac{\text{hours}}{\text{lb}}$. Roughly, it's how many extra hours it would take to cook a turkey that's 1 pound heavier. It should be positive, since it takes longer to cook a bigger bird.

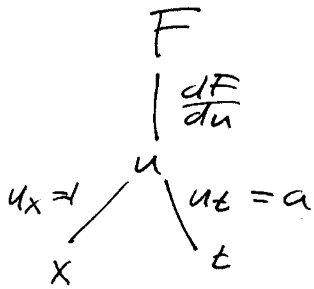
(c) (7 pts) Explain the physical meaning of the partial derivative $\partial z/\partial T$. What are the units? Do you expect it to be positive or negative? Why?

Units are $\frac{\text{hours}}{\text{degree F}}$. Roughly, it's how many extra hours it would take to cook the turkey if we turned the oven up 1 °F. It should be negative, since ~~it~~ the bird will cook faster at a higher temperature.

(4) (20 pts) Let $F(u)$ be a differentiable function of a single variable. Show that the function $w(x,t) = F(x + at)$ satisfies the one-dimensional wave equation

$$a^2 w_{xx} = w_{tt}.$$

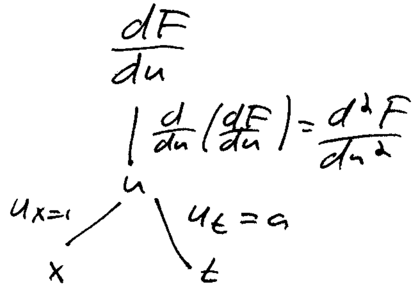
HINT: $w(x,t) = F(u)$, where $u = x + at$.



$$w_x = F_x = \frac{dF}{du} \cdot u_x = \frac{dF}{du}$$

So $w_{xx} = F_{xx} = \left(\frac{dF}{du}\right)_x$

Draw a new picture:



$$\left(\frac{dF}{du}\right)_x = \frac{d^2F}{du} \cdot u_x = \frac{d^2F}{du}$$

So $w_{xx} = \frac{d^2F}{du}.$

What's w_{tt}? Well, $w_t = F_t = \frac{dF}{du} \cdot u_t = a \cdot \frac{dF}{du}$

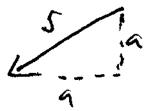
So $w_{tt} = F_{tt} = \left(a \frac{dF}{du}\right)_t = a \frac{d^2F}{du^2} \cdot u_t = a^2 \frac{d^2F}{du^2}.$

Thus ~~to~~ $a^2 w_{xx} = a^2 \frac{d^2F}{du} = w_{tt}$, and we're done.

(5) Let $g(x, y)$ be the depth (in inches) of mud at the point (x, y) in Portly Pig's pen. At a certain point in the pen, the greatest rate of depth increase, 5 inches per meter, is toward the southwest.

(a) If Portly Pig heads east from this point at a speed of 1/2 meter per second, how fast is the mud depth changing (inches per second)?

We know that ∇g has length 5 and ~~its~~ points south west, ie,

$\nabla g =$ . Thus $\nabla g = \frac{-5}{\sqrt{2}}\vec{i} - \frac{5}{\sqrt{2}}\vec{j}$. The directional derivative of depth g in the direction east is $\nabla g \cdot \vec{i} = \left(\frac{-5}{\sqrt{2}}\vec{i} - \frac{5}{\sqrt{2}}\vec{j}\right)\cdot(\vec{i})$ $= \frac{-5}{\sqrt{2}}$ in/m. Since Portly is moving at $1/2$ m/s, the time rate of change of depth is $\frac{-5}{\sqrt{2}}$ in/m $\cdot \, 1/2$ m/s $= \frac{-5}{2\sqrt{2}}$ in/s.

(b) If Portly wants to get out of the mud as quickly as possible, what direction should he head in?

$-\nabla g$ points in the direction of greatest decrease, so he should head northeast.