McWorter's Pentigree IFS Calculations

with(*LinearAlgebra*) :

Trig values at 36 and 72 degrees

$cos36 := \dfrac{(1 + \text{sqrt}(5))}{4}$

$$\frac{1}{4} + \frac{1}{4}\sqrt{5} \tag{1}$$

$sin36 := \text{sqrt}\left(\dfrac{(5 - \text{sqrt}(5))}{8}\right)$

$$\frac{1}{4}\sqrt{10 - 2\sqrt{5}} \tag{2}$$

$cos72 := \dfrac{(\text{sqrt}(5) - 1)}{4}$

$$\frac{1}{4}\sqrt{5} - \frac{1}{4} \tag{3}$$

$sin72 := 2 \cdot sin36 \cdot cos36$

$$\frac{1}{2}\sqrt{10 - 2\sqrt{5}}\left(\frac{1}{4} + \frac{1}{4}\sqrt{5}\right) \tag{4}$$

Scaling factor

$r := \dfrac{(3 - \text{sqrt}(5))}{2}$

$$\frac{3}{2} - \frac{1}{2}\sqrt{5} \tag{5}$$

Translation vectors

$P1 := combine(Vector(expand([r \cdot cos36, r \cdot sin36])), radical)$

$$\begin{bmatrix} \frac{1}{4}\sqrt{5} - \frac{1}{4} \\ \frac{3}{8}\sqrt{10 - 2\sqrt{5}} - \frac{1}{8}\sqrt{50 - 10\sqrt{5}} \end{bmatrix} \tag{6}$$

$evalf(P1)$

$$\begin{bmatrix} 0.3090169942 \\ 0.2245139882 \end{bmatrix} \tag{7}$$

$P2 := combine(P1 + Vector(expand([-r \cdot cos72, r \cdot sin72])), radical)$

$$\begin{bmatrix} -\dfrac{1}{4}\sqrt{5}+\dfrac{3}{4} \\[2mm] \dfrac{1}{4}\sqrt{10-2\sqrt{5}} \end{bmatrix} \tag{8}$$

evalf (%)

$$\begin{bmatrix} 0.1909830058 \\ 0.5877852522 \end{bmatrix} \tag{9}$$

$P3 := combine(P2 + Vector(expand([r \cdot cos36, -r \cdot sin36])), radical)$

$$\begin{bmatrix} \dfrac{1}{2} \\[2mm] -\dfrac{1}{8}\sqrt{10-2\sqrt{5}}+\dfrac{1}{8}\sqrt{50-10\sqrt{5}} \end{bmatrix} \tag{10}$$

evalf (%)

$$\begin{bmatrix} 0.5000000000 \\ 0.3632712641 \end{bmatrix} \tag{11}$$

$P4 := combine(P3 + Vector(expand([-r \cdot cos72, -r \cdot sin72])), radical)$

$$\begin{bmatrix} \dfrac{3}{2} - \dfrac{1}{2}\sqrt{5} \\[2mm] 0 \end{bmatrix} \tag{12}$$

evalf (%)

$$\begin{bmatrix} 0.381966012 \\ 0. \end{bmatrix} \tag{13}$$

$P5 := combine(P4 + Vector(expand([r \cdot cos36, -r \cdot sin36])), radical)$

$$\begin{bmatrix} -\dfrac{1}{4}\sqrt{5}+\dfrac{5}{4} \\[2mm] -\dfrac{3}{8}\sqrt{10-2\sqrt{5}}+\dfrac{1}{8}\sqrt{50-10\sqrt{5}} \end{bmatrix} \tag{14}$$

evalf (%)

$$\begin{bmatrix} 0.6909830058 \\ -0.2245139882 \end{bmatrix} \tag{15}$$

$P6 := combine(P5 + Vector(expand([r \cdot cos36, r \cdot sin36])), radical)$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{16}$$

(.... just to make sure we really do end up at the endpoint (1,0)!)