

Derivation of a closed form expression for  $\sum_{k=0}^{N-1} r^k \cos(k\theta)$

$$S := \text{Sum}(r^k \cdot \cos(k \cdot \text{theta}), k = 0 .. N - 1)$$

$$\sum_{k=0}^{N-1} r^k \cos(k\theta) \tag{1}$$

value(S)

$$-\frac{(\cos(\theta) r - 1) r^N \cos(N\theta)}{2 \cos(\theta) r - r^2 - 1} - \frac{r \sin(\theta) r^N \sin(N\theta)}{2 \cos(\theta) r - r^2 - 1} + \frac{\cos(\theta) r - 1}{2 \cos(\theta) r - r^2 - 1} \tag{2}$$

combine(%)

$$\frac{-r^{1+N} \cos(N\theta - \theta) + r^N \cos(N\theta) + \cos(\theta) r - 1}{2 \cos(\theta) r - r^2 - 1} \tag{3}$$