What was it you wanted?
The art of counting ballots

Jim Wiseman

Plurality voting

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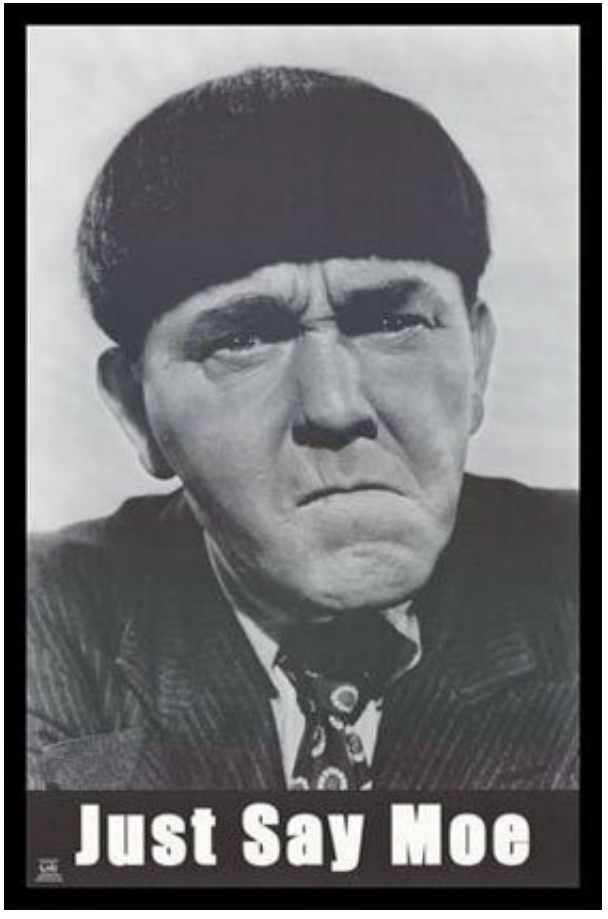
Moe wins!



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Moe wins!

If Curly drops out before the election, of course Moe will still win.

Plurality voting

Florida, 2000

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- Bush 49.18%
- Gore 49.17%
- Nader 1.64%

Plurality voting

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- Bush 49.18%
- Gore 49.17%
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But if Nader had dropped out, the results might have been

- Bush 49.5%
- Gore 50.5%

Plurality voting

The problem with plurality voting is that we're throwing away information.

We can't tell the difference between someone whose preferences are

$$(\text{Nader} \succ \text{Gore} \succ \text{Bush})$$

and someone else whose preferences are

$$(\text{Nader} \succ \text{Bush} \succ \text{Gore}).$$

Plurality voting

Analogy: Rank students according to how many A's they've gotten.

Then a student with 5 A's and 35 F's is ranked ahead of a student with 4 A's and 36 B's.

Mathematics

Each voter can rank the candidates from best to worst.

A *voting method* is a rule for looking at all the voters' individual preference lists, and combining them into one list: society's preferences.

(For example, plurality voting counts the number of times each candidate appears at the top of a voter's preference list.)

Mathematics

Mathematically speaking, a voting method is just a mapping from the space of all individuals' preferences to the space of societal preferences:

$$\{\text{all individuals' preferences}\} \to \{\text{societal preferences}\}.$$

If there are n different candidates, then there are $n!$ different ways to rank them (ignoring ties), so the set of preferences is the symmetric group S_n.
So if there are m voters, a voting method is a mapping

$$\overbrace{S_n \times S_n \times \cdots \times S_n}^{m \text{ times}} \to S_n.$$

(A kind of projection.)

Other voting methods

Example: 8 people prefer (Moe $\succ$ Larry $\succ$ Curly), 5 prefer (Curly $\succ$ Larry $\succ$ Moe), and 4 prefer (Larry $\succ$ Curly $\succ$ Moe).

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Borda count

Florida 2000

49 voters have (Gore $\succ$ Nader $\succ$ Bush),

48 voters have (Bush $\succ$ Nader $\succ$ Gore),

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3 voters have (Nader $\succ$ Gore $\succ$ Bush).

- Borda count gives Nader 103, Gore 101, and Bush 96.
- But if 3 of Gore's supporters strategically change their votes to (Gore $\succ$ Bush $\succ$ Nader), then we have Gore 101, Nader 100, and Bush 99.

Condorcet

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- *Maximum Happiness:* Make as many people as possible happy.
- *Minimum Unhappiness:* Make as few people as possible unhappy.

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- Theory developed by Steven Brams (political scientist) and Peter Fishburn (mathematician) in 1977.
- Lets us distinguish two kinds of (Moe $\succ$ Larry $\succ$ Curly) voters:
  - (Moe: *Yes*, Larry: *Yes*, Curly: *No*)
  - (Moe: *Yes*, Larry: *No*, Curly: *No*)

Approval voting

How do you vote?

Example: Just two candidates, Superman

Approval voting

How do you vote?

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Example: Just two candidates, Superman and Dr. Evil.

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- $1,000,000 worth of utility if Superman is elected;
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- $0.43 worth of utility if Mike Lynn is elected.

Average utility is $333,333.33, so Vote *Yes* on Superman, *No* on Dr. Evil, and *No* on Mike Lynn.

Approval voting

Subset elections

Say the results of an election using approval voting are:

$$(\text{Moe} \succ \text{Larry} \succ \text{Curly} \succ \text{Shemp})$$

What can we say about what the results would be if Shemp dropped out?

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Similarly, the election between just Moe and Larry could end up (Moe $\succ$ Larry) or (Larry $\succ$ Moe), regardless of the outcomes of 4- and 3-candidate elections.

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Proof: Cycles and symmetry.

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Example: We'll produce an election with outcome

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- Begin by creating *any* group of voters who give the outcome (Moe $\succ$ Larry $\succ$ Curly).
- Next, create an additional group V of voters: Voter A, Voter B, and Voter C.

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- And in the Moe-Curly election, A must vote (Moe: *Yes*, Curly: *No*).
- But in the Larry-Curly, he could vote either way. Choose (Larry: *No*, Curly: *Yes*).

Approval voting

Create Voter B and Voter C similarly to end up with

	(Moe, Larry, Curly)	(M, L)	(M, C)	(L, C)
Voter A	(Y,N,N)	(Y,N)	(Y,N)	(N,Y)
Voter B	(N,Y,N)	(N,Y)	(N,Y)	(Y,N)
Voter C	(N,N,Y)	(N,Y)	(N,Y)	(N,Y)

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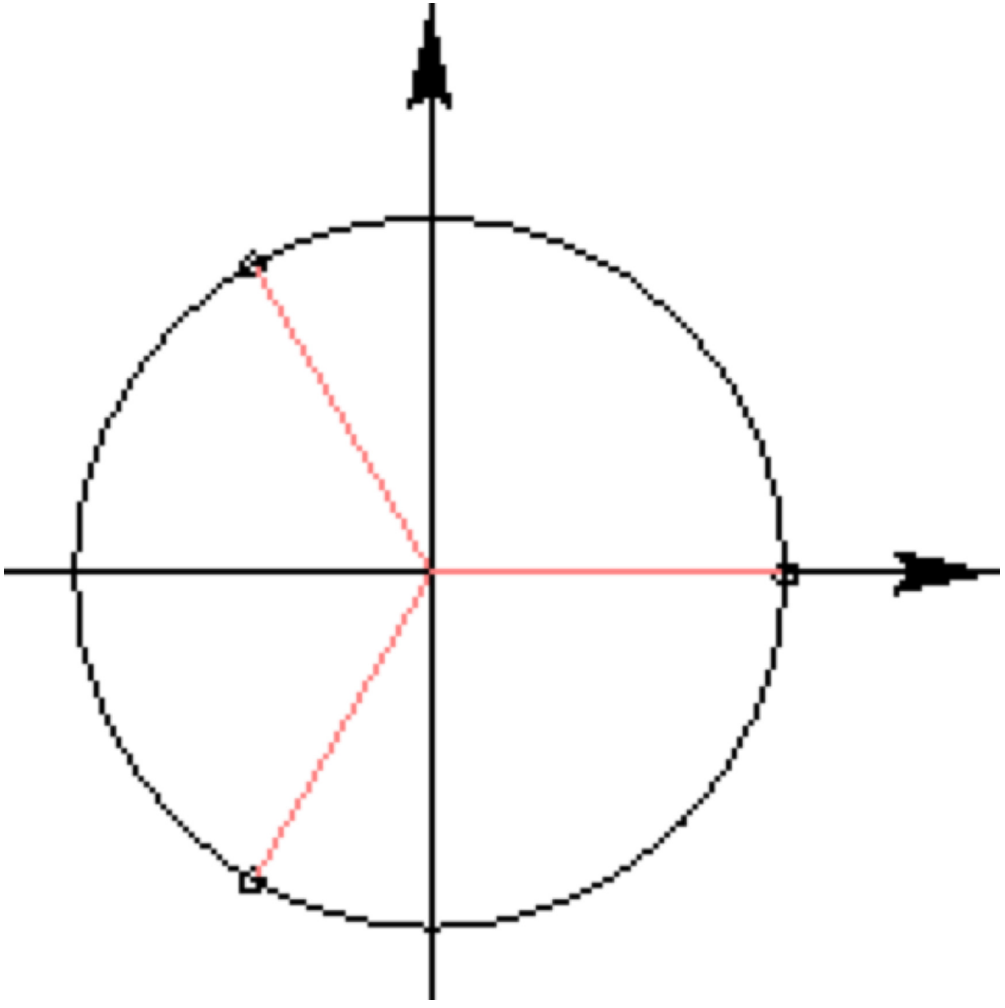
- The 2-way elections give the results we want.
- Add enough copies of V to our original group of voters, and we end up with the results we want.

Approval voting



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Proof: Linear algebra! (Have to show a certain matrix has maximal rank.)

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Example: 3 people vote (Moe: *Yes*, Larry: *Yes*, Curly: *No*); 2 people vote (Moe: *No*, Larry: ?, Curly: *Yes*).

Moe: 3/5, Larry: 3/3, Curly: 2/5, so Larry $\succ$ Moe $\succ$ Curly.

Modified approval voting

Example: Voters divided into two precincts.

1st precinct

#	(M,L,C)
10	(N,?,Y)
2	(N,Y,?)
3	(Y,N,N)
7	(Y,?,N)

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Moe 10/22 = 45%

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2nd precinct

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(Example of Simpson's paradox.)

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Arrow's Theorem: The only such system is a dictatorship.

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They're all the same for two-candidate elections.

References

- Brams and Fishburn, *Approval Voting*, 1982.
- Saari, *Basic Geometry of Voting*, 1995.
- Wiseman, Approval voting in subset elections, *Economic Theory* 15 (2000), no. 2, 477-483.