A Calculus Student Reads the Newspaper

Larry Riddle
Agnes Scott College

In the mid-1990's, John Allen Paulos wrote a book called *A Mathematician Reads the Newspaper*. When I teach calculus I like to get my students to read the newspaper for articles discussing ideas related to rates of change, in particular ideas that can be rephrased in terms of first and second derivatives. They pop up frequently, often in the business section, but sometimes even in the comics. I like to introduce these when we first start learning about the meaning of the second derivative and what it means, for example, to increase at an increasing rate. For each headline and/or article, I ask students to consider:

What is the function being discussed? What is the meaning of the variables?
Is the first derivative positive, negative, zero, or not possible to tell? What is the evidence?
Is the second derivative positive, negative, zero, or not possible to tell? What is the evidence?
What would a sketch of the graph look like?
Is the headline accurate?

What follows are some examples that I've collected over the years.

"The goal of a diet isn't losing weight, it's slowing down the gain"

Here is a *Garfield* comic from July 24, 1992.

(Used with permission of Universal Press Syndicate)

In class I ask students the following[1]:

Let $W(t)$ be Garfield's weight as a function of time. Rephrase Jon's last comment in terms of calculus by filling in each blank with one or more items from among the following expressions:

[1] My thanks to Evan Romer, Susquehanna Valley High School, for this reference and idea, posted to the AP Calculus Electronic Discussion Group on September 1, 1998.

$W < 0$ $W > 0$ $W' < 0$ $W' > 0$ $W'' < 0$ $W'' > 0$

"With Garfield, the goal of a diet isn't ________, it's _________."

Students must recognize that "losing weight" means that the weight function is decreasing ($W' < 0$, which for Garfield is *not* the goal.) Rather, Garfield's goal is to have the increase in weight slow down over time, a statement about the *second* derivative. One can imagine the graph of Garfield's weight being increasing but concave down ("slowing down the *gain*"). So the second blank should be $W'' < 0$ (or more accurately, $W' > 0$ and $W'' < 0$.)

Here is another comic that can be reinterpreted in terms of first and second derivatives:

(Used with permission of Chris Cassatt)

The first panel is the same as Garfield's goal for a diet. The second panel reflects a graph that is decreasing but concave up.

"Tech sector still strong, but growth decelerating"

Many of the newspaper articles that I have collected pertain to financial issues. One of the best was a column by Donald Ratajczak, at that time the director of the Economic Forecasting Center at Georgia State University, which appeared in the *Atlanta Journal-Constitution* on January 14, 1996. With a headline that proclaimed "Tech sector still strong, but growth decelerating," Ratajczak wrote that "Some years ago, one of my friends who is a stock analyst said I taught him the value of the 'second derivative.'" He went on to explain "The second derivative measures how much the change is changing. If the first [derivative] measures speed, the second measures acceleration. Thus, the second derivative measures whether the speed of technology growth is surging or slowing." His main point was an excellent example of why it is important to understand the relationship between tangent approximations and second derivatives:

"Wall Street analysts too often look through the rearview mirror. They see how fast speed has picked up and then project those gains forward…" In other words, the analysts perhaps use a tangent line approximation to project future data.

"They fail to note that the acceleration is slowing." The graph is increasing but concave down.

"Their earnings expectations are too high when earnings growth slows." The tangent line is above the concave down graph and thus provides an over-approximation.

He finished the column by noting that "[the semiconductor industry's book-to-bill ratio] measures the rate at which orders are growing relative to shipments. Orders are still growing above shipments, but the rate of growth is falling. And, thus, Wall Street hopes are being disappointed."

"The growth in low-carbohydrate products is slowing"

Sometimes an article will have enough data to allow additional investigation about the behavior of a first or second derivative. The headline above appeared in the *Atlanta Journal-Constitution* on August 18, 2004. It suggests that the graph of the low-carbohydrate sales function should be increasing and concave down. Accompanying the article was the following table (source: ACNielsen) showing the percentage change in sales of carb-conscious products, by month, compared to the previous month.

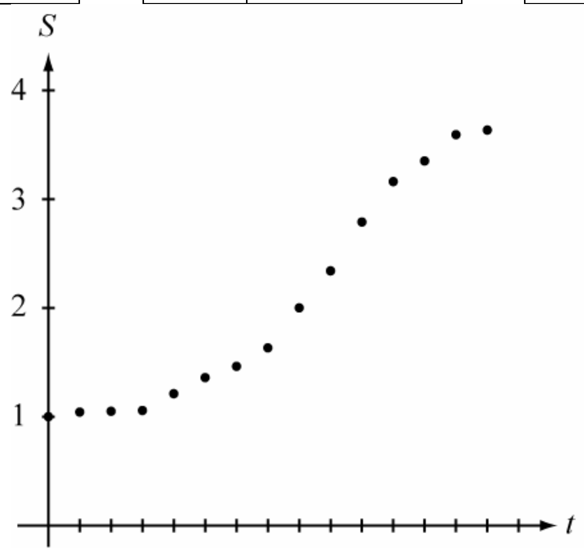
July 10, 2004	+1.2%
June 12, 2004	+7.2%
May 15, 2004	+6.0%
April 17, 2004	+13.3%
March 20, 2004	+19.2%
Feb. 21, 2004	+17.0%
Jan. 24, 2004	+22.5%
Dec. 27, 2003	+11.6%
Nov. 29, 2003	+7.6%
Nov. 1, 2003	+12.2%
Oct. 4, 2003	+14.6%
Sept. 6, 2003	+0.7%
Aug. 9, 2003	+0.7%
July 12, 2003	+4.3%

The first thing to realize is that you must read the table from the bottom up. The second is that the data is actually being measured every 28 days. The percentages start out more or less increasing as low-carb sales grow, but then the percentages do seem to decrease during 2004. But what do the percentages really measure? One interpretation is that if $S(t)$ represents the sales during "month" t (where a month is defined as 28 days), then the data in the table represents $r = \dfrac{S(t+1) - S(t)}{S(t)}$ and hence is an approximation to $\dfrac{S'(t)}{S(t)}$. Now $S(t+1) = (1+r)S(t)$, where r is *not* constant! Assuming that $S(0)$ represents 1 unit, we get the following values for the sales:

t	***S*(*t*)**
0	1
1	1.043
2	1.0503
3	1.0577
4	1.2121

t	***S*(*t*)**
5	1.36
6	1.4633
7	1.633
8	2.0005
9	2.3406

t	***S*(*t*)**
10	2.7899
11	3.161
12	3.3507
13	3.5919
14	3.635



This graph does support the conclusion of the headline that the growth in sales is slowing over the last 6 months (after a change in the graph from concave up to concave down.)

“Decline in AIDS deaths slows dramatically”

Not all graphs are increasing and concave down. The headline above appeared on CNN.com on August 30, 1999. The Associate Press article said that “The sharp decline in AIDS deaths that began two years ago with the development of powerful new drugs has suddenly been cut in half, raising questions about whether the medications are already losing their punch, health officials said Monday.” This is a good sentence for calculus students to translate into statements about the behavior of the first and second derivative. Here we presumably have a decreasing, concave up graph since the value of the (negative) derivative (“sharp decline”) has been “cut in half.” The concern for health officials, of course, is whether the graph might be approaching a local minimum.

“Slide of Internet stocks speeds up”

So proclaimed a headline in the *Atlanta Journal-Constitution* on August 5, 1999. “The big sell-off in the spring is suddenly accelerating, helping decimate the phenomenal gains achieved this year by many high-profile online companies.” This is a good example for students to appreciate the consequences of a decreasing, concave down stock graph!

“The increase in chip speed is accelerating, not slowing”

This was the headline for an article by John Markoff in the *New York Times* on February 4, 2002. In it, Markoff writes “Moore’s Law is the observation made in 1965 by the Intel co-founder Gordon Moore that the number of transistors on a chip – and so, approximately, the chip’s computing power – would continue to double roughly every 18 months.” This is a nice example for students to derive the formula of an exponential function, $C(t) = k(2)^{t/18}$, where $C(t)$ is the number of chips at time *t* in months. Such a function has a graph that is increasing and concave up. But Markoff goes on to say “But while Moore’s Law proved to be a remarkably accurate engineering forecast for three and a half decades, it is now apparent that chip speeds are doubling even more frequently than every 18 months.” So did the headline writer get it right? (Headlines are often written by someone other than the article’s author.) Moore’s Law would never predict a case in which the increase in chip speed would slow. Like models of exponential population growth, however, it was hard for many to believe that the number of transistors could continue to grow exponentially. Eventually a logistic type behavior might take over, causing the increase to begin slowing. According to this article, however, that has not been the case. In fact, as Markoff wrote in 2002, “That emphasis [on blinding computer speed] suggests that the trajectory of desktop PC performance increases of the last two years will not slow in the near future, but actually accelerate.” Hence the graph will continue to be concave up, with even larger second derivative values than predicted by Moore’s Law.

“Decline of ozone-harming chemicals suggests atmosphere may heal itself”

Good news! The amount of ozone-harming chemicals in the atmosphere is declining. At least that is the implication from this headline in the *Washington Post* on August 26, 1993. But the article summary reports that “The amount of ozone-destroying CFCs in the atmosphere, which had been rising rapidly for decades, suddenly slowed its rate of increase in 1989 and has nearly leveled off since then, NOAA scientists have found.” Oops. It is not the amount that has declined; it is the rate of increase that is declining. The graph must be still increasing but concave down, with a current slope that is nearly 0. That, however, is still good news.

“Torrid school enrollment rates cool off”

The *Atlanta Journal-Constitution* reported on January 1, 2000, that “The tremendous growth in Georgia’s public school enrollment has taken a breather. The number of new students is still increasing, only at a much slower rate.” The following data was included with the article:

Year	Enrollment	Increase	Percentage change
1994	1,270,948	--	--
1995	1,311,126	40,178	3.2%
1996	1,346,761	35,635	2.7%
1997	1,375,980	29,219	2.2%
1998	1,401,291	25,311	1.8%
1999	1,422,762	21,471	1.5%

Undoubtedly, the reader's attention was meant to be drawn to the last column. Yes, those percentage changes are really decreasing, so does that mean the headline is correct? But this is not the correct column to look at! The function of interest is $E(t)$, the enrollment in year *t*. The estimate of the derivative $E'(t)$ is given by the third column. Since those values are decreasing, it does appear reasonable to conclude that $E''(t)$ is negative, i.e. that the rate of increase is decreasing.

In fact, many newspaper headlines and articles are based on conclusions drawn from percentage changes, not actual changes. Consider, for example, this headline and paragraph from a 1994 College Board newsletter:

"College Board Survey Finds Tuition and Fees Increase at Slower Pace in 1993-94"

"For the third straight year the percentage increase at four-year colleges has fallen or stayed the same as the previous year," said Donald M. Steward, president of the College Board."

But percentage changes can, at times, be misleading. Consider, for example, the following table for $f(t) = t^2$ that mimics the setup of the table in the school enrollment example.

t	$f(t)$	Increase	Percentage change
1	1	--	--
2	4	3	300%
3	9	5	150%
4	16	7	77.8%
5	25	9	56.2%
6	36	11	44.0%

The percentage changes are decreasing whereas the actual changes are increasing. This data represents an increasing, concave *up* graph, not concave down. Now what would the table look like for $f(t) = t^2 + 100$?

t	$f(t)$	Increase	Percentage change
1	101	--	--
2	104	3	3.0%
3	109	5	4.8%
4	116	7	6.4%
5	125	9	7.8%
6	136	11	8.8%

This new graph has exactly the same shape as the previous one, but now the percentage changes increase.

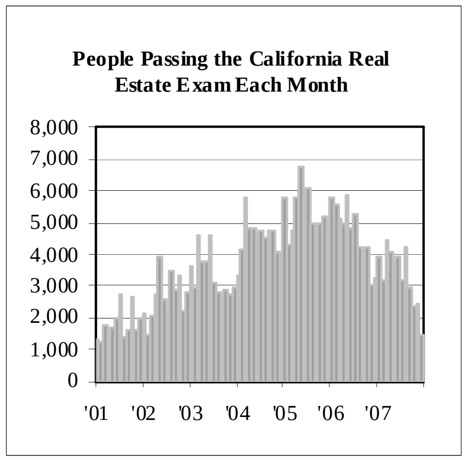
The quotient rule helps to explain what is happening. The percentage change over an interval of length 1 is an approximation to $\frac{f'}{f}$. Since

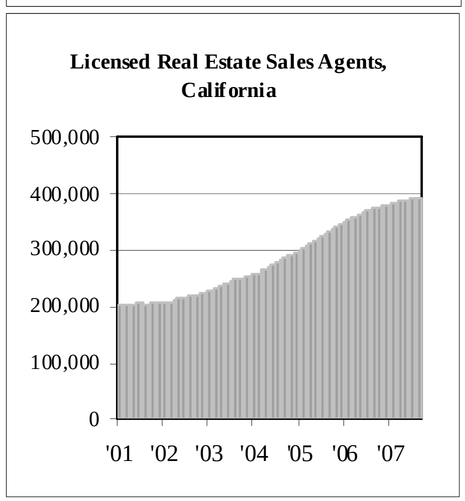
$$\left(\frac{f'}{f}\right)' = \frac{f''f - (f')^2}{f^2}$$

the relative size of f can influence whether the numerator is positive or negative. So conclusions about the concavity of the graph of f must be carefully made if the percentage changes are decreasing. It is likely, though, that if the values of f are much larger than the values of $|f'|$ (as one would expect in the College Board article about tuition increases), then f'' will also be negative if the percentage changes are decreasing. Notice, however, that if the percentage changes are increasing, then the quotient rule expression above indicates that f'' *must* be positive (or else the numerator is negative).

"**As Housing Market Cools, Far Fewer Become Agents**"

The September 7, 2007, business section of the *New York Times* carried an article about the effects of the housing market decline on real estate agents. A sidebar entitled "The Boom in Real Estate (Agents)" noted that "In California, once one of the hottest areas, fewer people are taking the [sales license] exam and the number of sales agents has leveled off."





The charts to the right were generated from data from the California Department of Real Estate web site (http://www.dre.ca.gov/gen_lic_exam_stats.html) covering the months from January 2001 to December 2007. If we assume that no real estate agent lost his or her license during this span, then the values in the top chart approximate the derivative of the function representing the values in the bottom chart. We see that the number of people passing the exam (and thus becoming a licensed real estate sales agent) has an overall increasing behavior from the beginning of 2001 to about the middle of 2005 and then has an overall decreasing behavior. Notice how this is reflected in the bottom graph which starts out concave up and then changes to concave down in the middle of 2005, right where the peak of the derivative graph would predict.

It does appear that the number of licensed real estate sales agents leveled off in 2007 as the article noted.

“**Mosquitoes on Tropical Island**”

Finally, we note that the types of behavior illustrated by many of these headlines have also appeared on the AP® Calculus Exam. Problem AB-2 on the 2004 Form B Exam presented students with a function modeling the rate of change of the number of mosquitoes on Tropical Island. Part (a) asked students to show that the number of mosquitoes is increasing at a specified time. Part (b) then asked “Is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate?” at this time. I can just imagine the headlines in the *Tropical Island Post-Gazette* after students solved this problem: “Mosquito infestation still growing, but at a slower rate!”

I have collected a large number of calculus-related headlines at my website

http://ecademy.agnesscott.edu/~lriddle/calculus

A nice project for calculus students is to look through the headlines and articles for examples suggesting each of the four basic possibilities for the shape of a graph (increasing/concave up; increasing/concave down; decreasing/concave up; and decreasing/concave down), and, for each example, to answer the questions I posed at the beginning of this article. An even better project is for students to find their own examples in the newspaper or online news articles. It is important to emphasize, however, that you are looking for headlines or articles that reflect on both the first *and* second derivatives.

Finally, if your students ever start to complain about having to learn about the product rule, quotient rule, or chain rule, you can always point them to the headline from the *Wall Street Journal* on November 23, 1993, Section C, page 1: “Congress is getting serious about the rules for derivatives”![2]

[2] Alas, not our kind of derivatives. These derivatives are financial contracts intended to provide companies with insurance against risks from changes in interest rates or currency exchange rates. Common examples are futures and options, whose values are tied to the prices of stocks and bonds. This article begins “Congress is taking action on its threats to legislate controls on the fast-growing and controversial market in financial derivatives, one of Wall Street’s fastest-growing products.”

Appendix

Here are some of the headlines at my website http://ecademy.agnesscott.edu/~lriddle/calculus. For copyright reasons, only brief excerpts of each article are included on the website. The paragraphs were chosen to illustrate the mathematical issues.

Credit woes rising, but at lower rate
Atlanta Journal-Constitution, October 30, 1991

CD yields continue decline, but at a reduced pace
Atlanta Journal-Constitution, April 24, 1991

First American continues losing, though not as badly
Atlanta Journal-Constitution, October 31, 1991

College Board Survey Finds Tuition and Fees Increase at Slower Pace in 1993-94
College Board Newsletter, 1993

Mortgage rates still climbing, but pace slows
Atlanta Journal-Constitution, April 8, 1994

Tech sector still strong, but growth decelerating
Atlanta Journal-Constitution, January 14, 1996

September figures show continued growth, though at a slower rate
CNNfn.com, November 3, 1998

Slide of Internet stocks speeds up
Atlanta Journal-Constitution, August 5, 1999

Inmate numbers rise, but at slower rate
The *Boston Globe*, August 16, 1999

Decline in AIDS deaths slows dramatically
CNN.com, August 30, 1999

Metro housing starts rise, but growth rate slowing
Atlanta Journal-Constitution, September 17, 1999

Torrid school enrollment rates cool off
Atlanta Journal-Constitution, January 1, 2000

U.S. gasoline price increase slowing, survey finds
CNN.com, June 25, 2000

U.S. growth expected to continue, but at slower pace
Atlanta Journal-Constitution, October 1, 2000

China growth rate slowing
Atlanta Journal-Constitution, March 28, 2001

U.S. kids getting fatter at faster rate
Atlanta Journal-Constitution, December 11, 2001

The Increase in Chip Speed is Accelerating, Not Slowing
The *New York Times*, February 4, 2002

Hartsfield's drop in passengers appears to have decelerated
Atlanta Journal-Constitution, April 17, 2003

The growth in low-carbohydrate products is slowing
Atlanta Journal-Constitution, August 18, 2004

College costs rise at slower rate
Associated Press (Yahoo! News), October 19 2004

Cell phone market growth slowing
Reuters (Yahoo! News), October 22, 2004

Rise in health spending slows
Atlanta Journal-Constitution, January 11, 2005

Gas prices dropping more slowly
USA Today, October 6, 2006

Cost of health insurance rises at reduced pace
Atlanta Journal Constitution, September 27, 2007

Economy still growing but at slower pace
Reuters, January 16, 2008