E) None of the above

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

A small country consists of four states. The population of State A is 67,200, the population of State B is 78,300, the population of State C is 73,800, and the population of State D is 80,700. The total number of seats in the legislature is 100.

1) The standard divisor is	1)
A) 1000.	
B) 10,000.	
C) 30,000.	
D) 3000.	
E) None of the above	
2) The standard quota for State C is	2)
A) 26.9.	
B) 25.7.	
C) 26.1.	
D) 24.6.	
E) None of the above	
,	
3) Under Hamilton's method, the apportionments to each state are	3)
A) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.	
B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.	
C) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.	
D) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.	
E) None of the above	
4) Using a divisor of $D = 2925$ , the modified quotas (to two decimal places) are	4)
A) State A: 22.40; State B: 26.10; State C: 24.60; State D: 26.90.	
B) State A: 22.58; State B: 26.67; State C: 24.93; State D: 27.28.	
C) State A: 22.97; State B: 26.77; State C: 25.23; State D: 27.59.	
D) State A: 22.74; State B: 26.86; State C: 25.12; State D: 27.43.	
E) None of the above	
5) Under Jefferson's method, the apportionments to each state are	5)
A) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.	
B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.	
C) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.	
D) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.	

3194	9066	4548	8192	
A	В	С	D	
		-	-	•
eats; State	D: 27 se	eats.		
•				
•				
•				
state are				8)
	D: 28 se	eats.		7)
ate D: 26.90	).			
ate D: 27.28	3.			
ate D: 26.48	3.			
ate D: 26.33	3.			
	ate D: 26.33 ate D: 26.43 ate D: 27.28 ate D: 26.90 ate are eats; State	ate D: 26.33. ate D: 26.48. ate D: 27.28. ate D: 26.90. ate are eats; State D: 28 se eats; State D: 27 se eats; State D: 26 se eats; State D: 26 se eats; State D: 27 se eats; State D: 28 se eats; State D: 28 se eats; State D: 26 se eats; State D: 27 se eats; State D: 28 se eats; State D: 28 se eats; State D: 26 se eats; State D: 27 se eats; State D: 27 se	ate D: 26.48.  ate D: 27.28.  ate D: 26.90.  ate are eats; State D: 28 seats. eats; State D: 27 seats. eats; State D: 26 seats. eats; State D: 26 seats. eats; State D: 27 seats. eats; State D: 27 seats. eats; State D: 27 seats. eats; State D: 28 seats. eats; State D: 28 seats. eats; State D: 26 seats. eats; State D: 27 seats.	ate D: 26.33. ate D: 26.48. ate D: 27.28. ate D: 26.90.  ate are eats; State D: 28 seats. eats; State D: 27 seats. eats; State D: 26 seats. eats; State D: 26 seats. eats; State D: 27 seats. eats; State D: 28 seats. eats; State D: 28 seats. eats; State D: 28 seats. eats; State D: 26 seats. eats; State D: 27 seats.

	Route	A	В	C	D	
	Daily average number of passengers	3194	9066	4548	8192	
9) The standard d	ivisor is					9)
A) 5000.						
B) 250.						
C) 500.						
D) 25,000.						
E) None of	the above					

e standard divisor represents
A) the daily average number of passengers per 50 buses.
B) the number of passengers that one bus is able to transport per day.
C) the daily average number of passengers per bus.
D) the number of buses required for 25,000 passengers.
E) None of the above

11) The standard quota of Route A (to 2 decimal places) is	11)
A) 7.14.	
B) 12.78.	
C) 6.39.	
D) 63.88.	
E) None of the above	
12) In process of applying Hamilton's method, the route receiving the "extra" bus is	12)
A) Route B.	
B) Route C.	
C) Route A.	
D) Route D.	
E) None of the above	
13) Find the apportionment of the buses among the routes using Hamilton's method. A) Route A: 7; Route B: 18; Route C: 9; Route D: 16	13)
B) Route A: 7; Route B: 18; Route C: 10; Route D: 16	
C) Route A: 6; Route B: 18; Route C: 9; Route D: 17	
D) Route A: 7; Route B: 17; Route C: 9; Route D: 17	
E) None of the above	
14) Find the apportionment of the buses among the routes using Jefferson's method.	14)
A) Route A: 7; Route B: 18; Route C: 10; Route D: 16	
B) Route A: 6; Route B: 18; Route C: 9; Route D: 17	
C) Route A: 7; Route B: 17; Route C: 9; Route D: 17	
D) Route A: 7; Route B: 18; Route C: 9; Route D: 16	
E) None of the above	
15) Find the apportionment of the buses among the routes using Adams' method.	15)
A) Route A: 7; Route B: 17; Route C: 9; Route D: 17	
B) Route A: 6; Route B: 18; Route C: 9; Route D: 17	
C) Route A: 7; Route B: 18; Route C: 9; Route D: 16	
D) Route A: 7; Route B: 18; Route C: 10; Route D: 16	
E) None of the above	
16) Find the apportionment of the buses among the routes using Webster's method.	16)
A) Route A: 7; Route B: 18; Route C: 10; Route D: 16	
B) Route A: 7; Route B: 17; Route C: 9; Route D: 17	
C) Route A: 7; Route B: 18; Route C: 9; Route D: 16	
D) Route A: 6; Route B: 18; Route C: 9; Route D: 17	
E) None of the above	

A country has four states. Suppose the population of State 1 is P<sub>1</sub>, the population of State 2 is P<sub>2</sub>, the population of State 3 is P<sub>3</sub>, and the population of State 4 is P<sub>4</sub>. Suppose also that the total number of seats in the legislature is M and the standard divisor is D.

17) The value of D is	17)
A) $P_1 + P_2 + P_3 + P_4$ .	
B) $\frac{P_1 \times P_2 \times P_3 \times P_4}{M}$ .	
M.	
C) $\frac{P_1 + P_2 + P_3 + P_4}{M}$ .	
$^{\prime}$ M	
D) $\frac{M}{P_1 + P_2 + P_3 + P_4}$ .	
E) None of the above	
18) If q <sub>1</sub> , q <sub>2</sub> , q <sub>3</sub> , and q <sub>4</sub> are the respective standard quotas for the four states, then	18)
q1 + q2 + q3 + q4 equals	
A) the number of seats in the legislature M.	
B) 0.	
C) the total population $P_1 + P_2 + P_3 + P_4$ .	
D) the standard divisor D.	
E) None of the above	
19) If J is the modified divisor used for Jefferson's method, then	19)
A) J can be less than, equal to, or greater than D.	′ <del></del>
B) J is always greater than or equal to D.	
C) J is always equal to D.	
D) J is always less than or equal to D.	
E) None of the above	
Solve the problem.	
20) Which of the following apportionment methods does not violate the quota rule?	20)
A) Adams' method	
B) Hamilton's method	
C) Jefferson's method	
D) Webster's method	
E) None of the above	
21) Which of the following apportionment methods can produce the Population paradox?	21)
A) Adams' method	, <u> </u>
B) Jefferson's method	
C) Webster's method	
D) Hamilton's method	

E) None of the above

22)	In a certain apportionment problem	, State X has	a standard	quota of	48.9.	The f	inal
	apportionment to State X is 50 seats	. This is calle	ed				

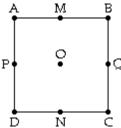
22) \_\_\_

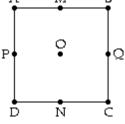
- A) an upper-quota violation.
- B) the population paradox.
- C) the Alabama paradox.
- D) a lower-quota violation.
- E) None of the above
- 23) A father wishes to distribute 16 pieces of candy among his 3 children (Abe, Betty, and Cindy) based on the number of hours each child spends doing chores around the house. Using a certain apportionment method, he has determined that Abe is to get 9 pieces of candy, Betty is to get 4 pieces, and Cindy is to get 3 pieces. However, just before he hands out the candy, he discovers that he has 17 pieces (not 16) of candy. When he apportions the 17 pieces of candy using the same apportionment method, Abe ends up with 10 pieces, Betty with 5 pieces, and Cindy with 2 pieces. This is an example of



- A) the new states paradox.
- B) the Alabama paradox.
- C) a violation of the quota rule.
- D) the population paradox.
- E) None of the above

The figure below is a square ABCD with center O. (M, N, P, and Q are the midpoints of the sides.)





24) Which of the following reflections is not a symmetry of the square?

24)

- A) the reflection with axis the line passing through A and C
- B) the reflection with axis the line passing through P and Q
- C) the reflection with axis the line passing through A and B
- D) the reflection with axis the line passing through M and N
- E) All of the above are symmetries of the square.

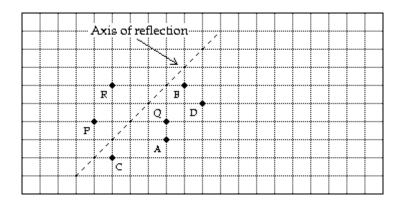
25)

- 25) Which of the following rotations is a symmetry of the square?
  - A) a 90° clockwise rotation with center P
  - B) a 90° clockwise rotation with center O
  - C) a 60° clockwise rotation with center O
  - D) a 90° clockwise rotation with center A
  - E) None of the above

26) Which of the following translations is a symmetry of the square?	26)
A) a translation that sends A to C	
B) a translation that sends A to B	
C) a translation that sends P to Q	
D) a translation that sends A to O	
E) None of the above	
27) The image of A under the reflection with axis the line passing through M and P is	27)
A) C.	
B) D.	
C) B.	
D) O.	
E) None of the above	
28) The image of A under a 90° clockwise rotation with center O is	28)
A) C.	
B) B.	
C) D.	
D) M.	
E) None of the above	
20) A topolation and a the point A to the point O. The image of B and on this topolation is	20)
29) A translation sends the point A to the point Q. The image of P under this translation is A) O.	29)
B) B.	
C) C.	
D) N.	
E) None of the above	
30) A glide reflection sends the point A to the point Q and the point P to the point C. The image of B	30)
under this glide reflection is	
A) P.	
B) D.	
C) A.	
D) N.	
E) None of the above	
31) A glide reflection sends the point A to the point Q and the point P to the point C. The axis of this	31)
glide reflection is a line passing through the points	
A) P and Q.	
B) M and N.	
C) A and B.	
D) A and Q.	
F) None of the above	

## Solve the problem.

- 32) A 7216° clockwise rotation is equivalent to
  - A) a 344° counterclockwise rotation.
  - B) a 376° clockwise rotation.
  - C) a 16° clockwise rotation.
  - D) All of the above
  - E) None of the above
- 33) A glide reflection having axis of reflection as shown below sends point P to point Q. The image of point R under this same glide reflection is



- A) A.
- B) B.
- C) C.
- D) D.
- E) None of the above
- 34) The letter C has a symmetry type
  - A) Z<sub>2</sub>.
  - B) D<sub>2</sub>.
  - C) D<sub>1</sub>.
  - D) Z<sub>1</sub>.
  - E) None of the above
- 35) The letter Q has a symmetry type
  - A) Z<sub>2</sub>.
  - B) D<sub>1</sub>.
  - C)  $Z_1$ .
  - D) D<sub>2</sub>.
  - E) None of the above

35) \_\_\_\_\_

34) \_\_\_\_\_

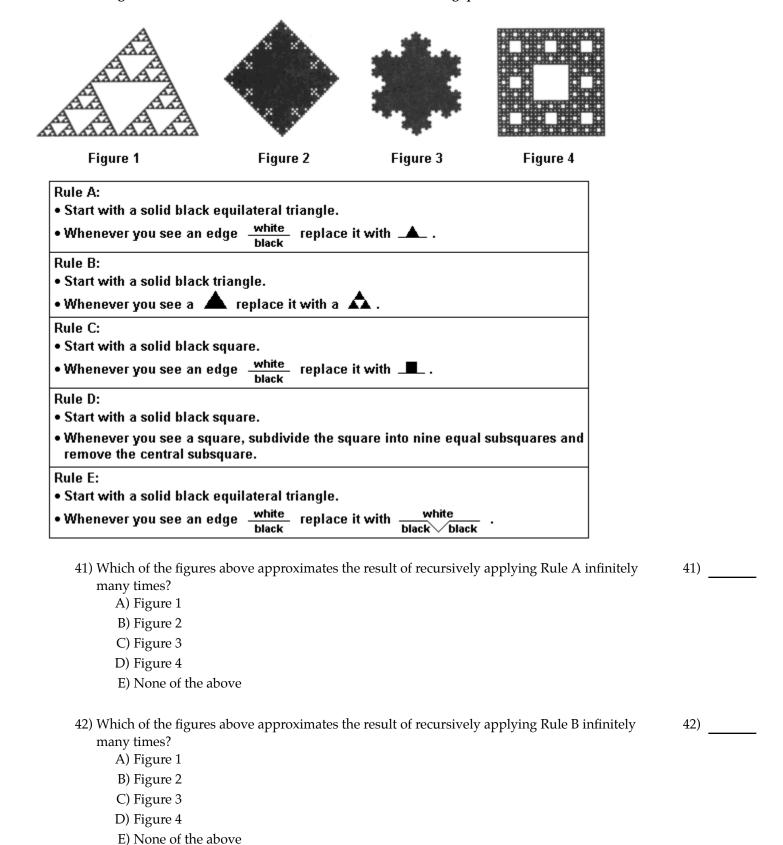
32) \_\_\_\_\_

33)

36) The letter Z has a symmetry type							36)
A) D <sub>2</sub> .							
B) Z <sub>1</sub> .							
C) D <sub>1</sub> .							
D) Z <sub>2</sub> .							
E) None of the above							
37) If an object has a 30° clockwise rot	ation	as one	e of it	s sym	metrie	es, then it must also have as a	a 37)
symmetry							
A) a 90° clockwise rotation.							
B) a 45° clockwise rotation.							
C) a translation.							
D) a reflection.							
E) None of the above							
38) The complete symmetries of the be		-	rn	ZZZ	ZZZZ	Z are the identity and	38)
A) translations and 180° rotations							
B) translations and 45° rotation		•	_				
C) translations and horizontal			•				
D) translations and vertical ref	lectior	ns onl	y.				
E) None of the above							
39) The complete symmetries of the be	order	pattei	rn	pbq	dpb	q d p b q d are the identit	ty and 39)
A) translations and glide reflec				1 1	1		,
B) translations and 180° rotations	ons on	ly.					
C) translations, glide reflection	s, and	l 180°	rotati	ions o	nly.		
D) translations only.							
E) None of the above							
40) The complete symmetries of the w	/allpa <sub>]</sub>	per pa	attern	show	n belo	ow are the identity and	40)
			:				
	В	В	В	В	В		
	В	В	В	В	В		
••	В.	В	В	В	в.	••	
	В	В	В	В	В		
	_		:				
			•				
A) translations and vertical ref	lectior	ns onl	y.				
B) translations and 180° rotations	ons on	ly.					
C) translations and horizontal reflections only.							

D) translations only.E) None of the above

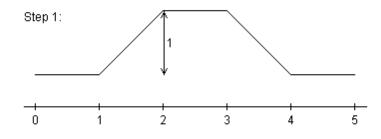
Refer to the figures and recursive rules below to answer the following question(s).

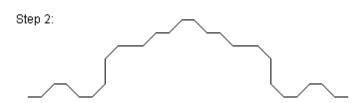


43) Which of the figures above approximates the result of recursively applying Rule C infinitely many times?	43) _	
A) Figure 1		
B) Figure 2		
C) Figure 3		
D) Figure 4		
E) None of the above		
44) Which of the figures above approximates the result of recursively applying Rule D infinitely many times?	44) _	
A) Figure 1		
B) Figure 2		
C) Figure 3		
D) Figure 4		
E) None of the above		
45) Which of the figures above approximates the result of recursively applying Rule E infinitely many times?	45) _	
A) Figure 1		
B) Figure 2		
C) Figure 3		
D) Figure 4		
E) None of the above		
Solve the problem.		
46) If the area of the starting triangle in the construction of the Koch snowflake is 5, then the area of the Koch snowflake is	46) _	
A) 8.		
B) 10.		
C) infinite.		
D) 0.		
E) None of the above		
47) Suppose that the perimeter of the starting triangle in the construction of the Koch snowflake is 5. Then the length of the boundary of the Koch snowflake is	47) _	
A) 0.		
B) infinite.		
C) 10.		
D) 8.		
E) None of the above		

The following question(s) refer to a fractal defined by the recursive procedure :

- Start with a line segment of length 5.
- Step 1: Replace the line segment with \_\_\_\_\_ (see figure below).
- Step 2: Replace each line segment in the previous figure with \_\_\_\_\_ (see figure below).
- Step 3, 4, 5, etc.: Replace each line segment in the previous figure with \_\_\_\_\_\_.





48) What is the length of the figure at step 1 of the construction?

48)

- A) 5
- B) 3 +  $2\sqrt{2}$
- C)  $5\sqrt{2}$
- D)  $7 + 2\sqrt{2}$
- E) None of the above
- 49) How many square units of area are added above the original horizontal line segment at step 1 of 49) \_\_\_\_\_ the construction?
  - A) 4
  - B) 1
  - C) 3
  - D) 2
  - E) None of the above
- 50) How many line segments appear in step 2 of the construction?

50)

- A) 52
- B) 5
- C) 25
- D)  $5 \times 2$
- E) None of the above

51) How many line segments appear in step 4 of the construction?

51) \_\_\_\_\_

- A)  $5 \times 4$
- B) 45
- C) 5
- D) 54
- E) None of the above

52) What is the length of the leftmost line segment in step 3 of the construction?

52)

- A)  $\left(\frac{1}{5}\right)^3$
- $B)\left(\frac{1}{5}\right)^2$
- C)  $\frac{1}{3}$
- $D)\left(\frac{1}{3}\right)^5$
- E) None of the above

Solve the problem.

53) If a = 1 + i and b = i, then ab = i

53) \_\_\_\_\_

54) \_\_\_\_

- A) -1 + i.
- B) 2.
- C) 1 + i.
- D) 0.
- E) None of the above

To answer the following question(s), refer to the Mandelbrot replacement process described by:

• Start: Choose an arbitrary complex number s, called the seed of the Mandelbrot sequence. Set the seed s to be the initial term of the sequence ( $s_0 = s$ ).

• Procedure M: To find the next term in the sequence, square the preceding term and add the seed ( $s_{N+1} = s_N^2 + s$ ).

54) For the seed s = 2, the second and third values of the sequence ( $s_1$  and  $s_2$ ) are given by

- A) 6 and 42.
- B) 2 and 2.
- C) 4 and 16.
- D) 6 and 38.
- E) None of the above

55) For the seed $s = -3$ , the Mandelbrot replacement process	55)	
A) gives values that have no pattern.		
B) goes off to infinity.		
C) is periodic.		
D) gives values that get closer and closer to -1.		
E) None of the above		
56) Suppose that when we apply the Mandelbrot replacement process we get $s_6 = 2$ and $s_7 = 2$ . Then	56)	
the seed s is		
A) $s = -1$ .		
B) $s = 2$ .		
C) $s = 1$ .		
D) $s = -2$ .		
E) None of the above		
57) If we apply the Mandelbrot replacement process to the seed $s=2i$ , then $s_1=$	57)	
A) -4.		
B) -4 + 2i.		
C) 4i.		
D) -2i.		
E) None of the above		
Solve the problem.		
58) Of the following objects in nature, which one could never have symmetry of scale?	58) _	
A) a mountain		
B) a coastline		
C) a soap bubble		
D) a cloud		
E) All of the above could have symmetry of scale.		
59) Which of the following objects has exact symmetry of scale?	59)	
A) the Koch snowflake		
B) a tree		
C) the Mandelbrot set		
D) a head of cauliflower		

E) None of the above

## Answer Key

Testname: 101PRACMT2

- 1) D
- 2) D
- 3) A
- 4) C
- 5) C
- 6) A
- 7) B
- 8) A
- 9) C
- 10) C
- 11) C
- 12) C
- 13) A
- 14) B
- 15) C
- 16) D
- 17) C
- 18) A
- 19) D
- 20) B
- 21) D
- 22) A
- 23) B
- 24) C
- 25) B
- 26) E
- 27) D
- 28) B
- 29) C
- 30) A
- 31) B
- 32) D
- 33) D
- 34) C
- 35) C
- 36) D
- 37) A
- 38) A
- 39) B
- 40) C
- 41) C
- 42) A 43) B
- 44) D
- 45) E
- 46) A
- 47) B
- 48) B
- 49) D

Answer Key Testname: 101PRACMT2

- 50) A 51) D
- 52) B
- 53) A 54) D
- 55) B
- 56) D
- 57) B
- 58) C 59) A