MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

A small country consists of four states. The population of State A is 67,200, the population of State B is 78,300, the population of State C is 73,800, and the population of State D is 80,700. The total number of seats in the legislature is 100.

1) The standard divisor is
   A) 1000.
   B) 10,000.
   C) 30,000.
   D) 3000.
   E) None of the above

2) The standard quota for State C is
   A) 26.9.
   B) 25.7.
   C) 26.1.
   D) 24.6.
   E) None of the above

3) Under Hamilton’s method, the apportionments to each state are
   A) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
   B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
   C) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
   D) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
   E) None of the above

4) Using a divisor of D = 2925, the modified quotas (to two decimal places) are
   A) State A: 22.40; State B: 26.10; State C: 24.60; State D: 26.90.
   B) State A: 22.58; State B: 26.67; State C: 24.93; State D: 27.28.
   C) State A: 22.97; State B: 26.77; State C: 25.23; State D: 27.59.
   D) State A: 22.74; State B: 26.86; State C: 25.12; State D: 27.43.
   E) None of the above

5) Under Jefferson’s method, the apportionments to each state are
   A) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
   B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
   C) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
   D) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
   E) None of the above
6) Using a divisor of $D = 3065$ the modified quotas (to 2 decimal places) are
   A) State A: 21.92; State B: 25.55; State C: 24.08; State D: 26.33.
   B) State A: 21.94; State B: 25.86; State C: 24.12; State D: 26.48.
   C) State A: 22.58; State B: 26.67; State C: 24.93; State D: 27.28.
   D) State A: 22.40; State B: 26.10; State C: 24.60; State D: 26.90.
   E) None of the above

7) Under Adams’ method the apportionments to each state are
   A) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
   B) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
   C) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
   D) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
   E) None of the above

8) Under Webster’s method the apportionments to each state are
   A) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
   B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
   C) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
   D) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
   E) None of the above

A bus company operates four bus routes (A, B, C, and D) and 50 buses. The buses are apportioned among the routes on the basis of average number of daily passengers per route which is given in the following table.

<table>
<thead>
<tr>
<th>Route</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily average number of passengers</td>
<td>3194</td>
<td>9066</td>
<td>4548</td>
<td>8192</td>
</tr>
</tbody>
</table>

9) The standard divisor is
   A) 5000.
   B) 250.
   C) 500.
   D) 25,000.
   E) None of the above

10) The standard divisor represents
    A) the daily average number of passengers per 50 buses.
    B) the number of passengers that one bus is able to transport per day.
    C) the daily average number of passengers per bus.
    D) the number of buses required for 25,000 passengers.
    E) None of the above
11) The standard quota of Route A (to 2 decimal places) is
   A) 7.14.
   B) 12.78.
   C) 6.39.
   D) 63.88.
   E) None of the above

12) In process of applying Hamilton’s method, the route receiving the “extra” bus is
   A) Route B.
   B) Route C.
   C) Route A.
   D) Route D.
   E) None of the above

13) Find the apportionment of the buses among the routes using Hamilton’s method.
   A) Route A: 7; Route B: 18; Route C: 9; Route D: 16
   B) Route A: 7; Route B: 18; Route C: 10; Route D: 16
   C) Route A: 6; Route B: 18; Route C: 9; Route D: 17
   D) Route A: 7; Route B: 17; Route C: 9; Route D: 17
   E) None of the above

14) Find the apportionment of the buses among the routes using Jefferson’s method.
   A) Route A: 7; Route B: 18; Route C: 10; Route D: 16
   B) Route A: 6; Route B: 18; Route C: 9; Route D: 17
   C) Route A: 7; Route B: 17; Route C: 9; Route D: 17
   D) Route A: 7; Route B: 18; Route C: 9; Route D: 16
   E) None of the above

15) Find the apportionment of the buses among the routes using Adams’ method.
   A) Route A: 7; Route B: 17; Route C: 9; Route D: 17
   B) Route A: 6; Route B: 18; Route C: 9; Route D: 17
   C) Route A: 7; Route B: 18; Route C: 9; Route D: 16
   D) Route A: 7; Route B: 18; Route C: 10; Route D: 16
   E) None of the above

16) Find the apportionment of the buses among the routes using Webster’s method.
   A) Route A: 7; Route B: 18; Route C: 10; Route D: 16
   B) Route A: 7; Route B: 17; Route C: 9; Route D: 17
   C) Route A: 7; Route B: 18; Route C: 9; Route D: 16
   D) Route A: 6; Route B: 18; Route C: 9; Route D: 17
   E) None of the above
A country has four states. Suppose the population of State 1 is \( P_1 \), the population of State 2 is \( P_2 \), the population of State 3 is \( P_3 \), and the population of State 4 is \( P_4 \). Suppose also that the total number of seats in the legislature is \( M \) and the standard divisor is \( D \).

17) The value of \( D \) is
   A) \( P_1 + P_2 + P_3 + P_4 \).
   B) \( \frac{P_1 \times P_2 \times P_3 \times P_4}{M} \).
   C) \( \frac{P_1 + P_2 + P_3 + P_4}{M} \).
   D) \( \frac{M}{P_1 + P_2 + P_3 + P_4} \).
   E) None of the above

18) If \( q_1, q_2, q_3, \) and \( q_4 \) are the respective standard quotas for the four states, then
   \( q_1 + q_2 + q_3 + q_4 \) equals
   A) the number of seats in the legislature \( M \).
   B) 0.
   C) the total population \( P_1 + P_2 + P_3 + P_4 \).
   D) the standard divisor \( D \).
   E) None of the above

19) If \( J \) is the modified divisor used for Jefferson’s method, then
   A) \( J \) can be less than, equal to, or greater than \( D \).
   B) \( J \) is always greater than or equal to \( D \).
   C) \( J \) is always equal to \( D \).
   D) \( J \) is always less than or equal to \( D \).
   E) None of the above

Solve the problem.
20) Which of the following apportionment methods does not violate the quota rule?
   A) Adams’ method
   B) Hamilton’s method
   C) Jefferson’s method
   D) Webster’s method
   E) None of the above

21) Which of the following apportionment methods can produce the Population paradox?
   A) Adams’ method
   B) Jefferson’s method
   C) Webster’s method
   D) Hamilton’s method
   E) None of the above
22) In a certain apportionment problem, State X has a standard quota of 48.9. The final apportionment to State X is 50 seats. This is called
   A) an upper-quota violation.
   B) the population paradox.
   C) the Alabama paradox.
   D) a lower-quota violation.
   E) None of the above

23) A father wishes to distribute 16 pieces of candy among his 3 children (Abe, Betty, and Cindy) based on the number of hours each child spends doing chores around the house. Using a certain apportionment method, he has determined that Abe is to get 9 pieces of candy, Betty is to get 4 pieces, and Cindy is to get 3 pieces. However, just before he hands out the candy, he discovers that he has 17 pieces (not 16) of candy. When he apportions the 17 pieces of candy using the same apportionment method, Abe ends up with 10 pieces, Betty with 5 pieces, and Cindy with 2 pieces. This is an example of
   A) the new states paradox.
   B) the Alabama paradox.
   C) a violation of the quota rule.
   D) the population paradox.
   E) None of the above

The figure below is a square ABCD with center O. (M, N, P, and Q are the midpoints of the sides.)

24) Which of the following reflections is not a symmetry of the square?
   A) the reflection with axis the line passing through A and C
   B) the reflection with axis the line passing through P and Q
   C) the reflection with axis the line passing through A and B
   D) the reflection with axis the line passing through M and N
   E) All of the above are symmetries of the square.

25) Which of the following rotations is a symmetry of the square?
   A) a 90° clockwise rotation with center P
   B) a 90° clockwise rotation with center O
   C) a 60° clockwise rotation with center O
   D) a 90° clockwise rotation with center A
   E) None of the above
26) Which of the following translations is a symmetry of the square?
   A) a translation that sends A to C
   B) a translation that sends A to B
   C) a translation that sends P to Q
   D) a translation that sends A to O
   E) None of the above

27) The image of A under the reflection with axis the line passing through M and P is
   A) C.
   B) D.
   C) B.
   D) O.
   E) None of the above

28) The image of A under a 90° clockwise rotation with center O is
   A) C.
   B) B.
   C) D.
   D) M.
   E) None of the above

29) A translation sends the point A to the point Q. The image of P under this translation is
   A) O.
   B) B.
   C) C.
   D) N.
   E) None of the above

30) A glide reflection sends the point A to the point Q and the point P to the point C. The image of B
    under this glide reflection is
    A) P.
    B) D.
    C) A.
    D) N.
    E) None of the above

31) A glide reflection sends the point A to the point Q and the point P to the point C. The axis of this
    glide reflection is a line passing through the points
    A) P and Q.
    B) M and N.
    C) A and B.
    D) A and Q.
    E) None of the above
Solve the problem.

32) A 7216° clockwise rotation is equivalent to
   A) a 344° counterclockwise rotation.
   B) a 376° clockwise rotation.
   C) a 16° clockwise rotation.
   D) All of the above
   E) None of the above

33) A glide reflection having axis of reflection as shown below sends point P to point Q. The image of point R under this same glide reflection is

![Diagram showing glide reflection]

A) A.
B) B.
C) C.
D) D.
E) None of the above

34) The letter C has a symmetry type
   A) $Z_2$.
   B) $D_2$.
   C) $D_1$.
   D) $Z_1$.
   E) None of the above

35) The letter Q has a symmetry type
   A) $Z_2$.
   B) $D_1$.
   C) $Z_1$.
   D) $D_2$.
   E) None of the above
36) The letter Z has a symmetry type
   A) D2.
   B) Z1.
   C) D1.
   D) Z2.
   E) None of the above

37) If an object has a 30° clockwise rotation as one of its symmetries, then it must also have as a symmetry
   A) a 90° clockwise rotation.
   B) a 45° clockwise rotation.
   C) a translation.
   D) a reflection.
   E) None of the above

38) The complete symmetries of the border pattern \ldots Z Z Z Z Z Z \ldots are the identity and
   A) translations and 180° rotations only.
   B) translations and 45° rotations only.
   C) translations and horizontal reflections only.
   D) translations and vertical reflections only.
   E) None of the above

39) The complete symmetries of the border pattern \ldots p b q d p b q d p b q d \ldots are the identity and
   A) translations and glide reflections only.
   B) translations and 180° rotations only.
   C) translations, glide reflections, and 180° rotations only.
   D) translations only.
   E) None of the above

40) The complete symmetries of the wallpaper pattern shown below are the identity and

```
  B   B   B   B   B
  B   B   B   B   B
  ... B   B   B   B   B
  B   B   B   B   B
  ...
```

   A) translations and vertical reflections only.
   B) translations and 180° rotations only.
   C) translations and horizontal reflections only.
   D) translations only.
   E) None of the above
Refer to the figures and recursive rules below to answer the following question(s).

**Rule A:**
- Start with a solid black equilateral triangle.
- Whenever you see an edge replace it with .

**Rule B:**
- Start with a solid black triangle.
- Whenever you see a replace it with .

**Rule C:**
- Start with a solid black square.
- Whenever you see an edge replace it with .

**Rule D:**
- Start with a solid black square.
- Whenever you see a square, subdivide the square into nine equal subsquares and remove the central subsquare.

**Rule E:**
- Start with a solid black equilateral triangle.
- Whenever you see an edge replace it with .

41) Which of the figures above approximates the result of recursively applying Rule A infinitely many times?
   A) Figure 1
   B) Figure 2
   C) Figure 3
   D) Figure 4
   E) None of the above

42) Which of the figures above approximates the result of recursively applying Rule B infinitely many times?
   A) Figure 1
   B) Figure 2
   C) Figure 3
   D) Figure 4
   E) None of the above
43) Which of the figures above approximates the result of recursively applying Rule C infinitely many times?
   A) Figure 1
   B) Figure 2
   C) Figure 3
   D) Figure 4
   E) None of the above

44) Which of the figures above approximates the result of recursively applying Rule D infinitely many times?
   A) Figure 1
   B) Figure 2
   C) Figure 3
   D) Figure 4
   E) None of the above

45) Which of the figures above approximates the result of recursively applying Rule E infinitely many times?
   A) Figure 1
   B) Figure 2
   C) Figure 3
   D) Figure 4
   E) None of the above

Solve the problem.
46) If the area of the starting triangle in the construction of the Koch snowflake is 5, then the area of the Koch snowflake is
   A) 8.
   B) 10.
   C) infinite.
   D) 0.
   E) None of the above

47) Suppose that the perimeter of the starting triangle in the construction of the Koch snowflake is 5. Then the length of the boundary of the Koch snowflake is
   A) 0.
   B) infinite.
   C) 10.
   D) 8.
   E) None of the above
The following question(s) refer to a fractal defined by the recursive procedure:

- Start with a line segment of length 5.
- Step 1: Replace the line segment with \[\text{—} \quad \text{—}\] (see figure below).
- Step 2: Replace each line segment in the previous figure with \[\text{—} \quad \text{—}\] (see figure below).
- Step 3, 4, 5, etc.: Replace each line segment in the previous figure with \[\text{—} \quad \text{—}\] .

Step 1:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\downarrow & & & & & \\
& & 1 & & & \\
\end{array}
\]

Step 2:

48) What is the length of the figure at step 1 of the construction?  
A) 5  
B) \(3 + 2\sqrt{2}\)  
C) \(5\sqrt{2}\)  
D) \(7 + 2\sqrt{2}\)  
E) None of the above

49) How many square units of area are added above the original horizontal line segment at step 1 of the construction?  
A) 4  
B) 1  
C) 3  
D) 2  
E) None of the above

50) How many line segments appear in step 2 of the construction?  
A) \(5^2\)  
B) 5  
C) 25  
D) \(5 \times 2\)  
E) None of the above
51) How many line segments appear in step 4 of the construction?  
   A) $5 \times 4$  
   B) 45  
   C) 5  
   D) 54  
   E) None of the above

52) What is the length of the leftmost line segment in step 3 of the construction?  
   A) $\left(\frac{1}{5}\right)^3$  
   B) $\left(\frac{1}{5}\right)^2$  
   C) $\frac{1}{3}$  
   D) $\left(\frac{1}{3}\right)^3$  
   E) None of the above

Solve the problem.

53) If $a = 1 + i$ and $b = i$, then $ab =$  
   A) $-1 + i$  
   B) 2.  
   C) $1 + i$.  
   D) 0.  
   E) None of the above

To answer the following question(s), refer to the Mandelbrot replacement process described by:

- Start: Choose an arbitrary complex number $s$, called the seed of the Mandelbrot sequence. Set the seed $s$ to be the initial term of the sequence ($s_0 = s$).

- Procedure M: To find the next term in the sequence, square the preceding term and add the seed ($s_{N+1} = s^2_N + s$).

54) For the seed $s = 2$, the second and third values of the sequence ($s_1$ and $s_2$) are given by  
   A) 6 and 42.  
   B) 2 and 2.  
   C) 4 and 16.  
   D) 6 and 38.  
   E) None of the above
55) For the seed $s = -3$, the Mandelbrot replacement process

A) gives values that have no pattern.
B) goes off to infinity.
C) is periodic.
D) gives values that get closer and closer to $-1$.
E) None of the above

56) Suppose that when we apply the Mandelbrot replacement process we get $s_6 = 2$ and $s_7 = 2$. Then the seed $s$ is

A) $s = -1$.
B) $s = 2$.
C) $s = 1$.
D) $s = -2$.
E) None of the above

57) If we apply the Mandelbrot replacement process to the seed $s = 2i$, then $s_1 =$

A) $-4$.
B) $-4 + 2i$.
C) $4i$.
D) $-2i$.
E) None of the above

**Solve the problem.**

58) Of the following objects in nature, which one could never have symmetry of scale?

A) a mountain
B) a coastline
C) a soap bubble
D) a cloud
E) All of the above could have symmetry of scale.

59) Which of the following objects has exact symmetry of scale?

A) the Koch snowflake
B) a tree
C) the Mandelbrot set
D) a head of cauliflower
E) None of the above
1) D
2) D
3) A
4) C
5) C
6) A
7) B
8) A
9) C
10) C
11) C
12) C
13) A
14) B
15) C
16) D
17) C
18) A
19) D
20) B
21) D
22) A
23) B
24) C
25) B
26) E
27) D
28) B
29) C
30) A
31) B
32) D
33) D
34) C
35) C
36) D
37) A
38) A
39) B
40) C
41) C
42) A
43) B
44) D
45) E
46) A
47) B
48) B
49) D
Answer Key
Testname: 101PRACMT2

50) A
51) D
52) B
53) A
54) D
55) B
56) D
57) B
58) C
59) A