MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.
A small country consists of four states. The population of State $A$ is 67,200 , the population of State $B$ is 78,300 , the population of State $C$ is 73,800 , and the population of State $D$ is 80,700 . The total number of seats in the legislature is 100.

1) The standard divisor is
A) 1000 .
B) 10,000 .
C) 30,000.
D) 3000 .
E) None of the above
2) The standard quota for State $C$ is
A) 26.9 .
B) 25.7 .
C) 26.1 .
D) 24.6 .
E) None of the above
3) Under Hamilton's method, the apportionments to each state are
A) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
C) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
D) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
E) None of the above
4) Using a divisor of $D=2925$, the modified quotas (to two decimal places) are
A) State A: 22.40; State B: 26.10; State C: 24.60; State D: 26.90.
B) State A: 22.58; State B: 26.67; State C: 24.93; State D: 27.28.
C) State A: 22.97; State B: 26.77; State C: 25.23; State D: 27.59 .
D) State A: 22.74; State B: 26.86; State C: 25.12; State D: 27.43 .
E) None of the above
5) Under Jefferson's method, the apportionments to each state are
A) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
C) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
D) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
E) None of the above
6) $\qquad$
7) $\qquad$
8) $\qquad$
9) $\qquad$
10) $\qquad$
11) Using a divisor of $\mathrm{D}=3065$ the modified quotas (to 2 decimal places) are
12) $\qquad$
A) State A: 21.92; State B: 25.55 ; State C: 24.08 ; State D: 26.33 .
B) State A: 21.94; State B: 25.86; State C: 24.12 ; State D: 26.48 .
C) State A: 22.58; State B: 26.67; State C: 24.93; State D: 27.28 .
D) State A: 22.40; State B: 26.10; State C: 24.60 ; State D: 26.90 .
E) None of the above
13) Under Adams' method the apportionments to each state are
14) $\qquad$
A) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
B) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
C) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
D) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
E) None of the above
15) Under Webster's method the apportionments to each state are
16) $\qquad$
A) State A: 22 seats; State B: 26 seats; State C: 25 seats; State D: 27 seats.
B) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 28 seats.
C) State A: 22 seats; State B: 26 seats; State C: 24 seats; State D: 26 seats.
D) State A: 23 seats; State B: 26 seats; State C: 24 seats; State D: 27 seats.
E) None of the above

## A bus company operates four bus routes (A, B, C, and D) and 50 buses. The buses are apportioned among the routes on the basis of average number of daily passengers per route which is given in the following table.

| Route | A | B | C | D |
| :--- | :---: | :---: | :---: | :---: |
| Daily average number of passengers | 3194 | 9066 | 4548 | 8192 |

9) The standard divisor is
A) 5000 .
B) 250 .
C) 500 .
D) 25,000 .
E) None of the above
10) The standard divisor represents
11) 

A) the daily average number of passengers per 50 buses.
B) the number of passengers that one bus is able to transport per day.
C) the daily average number of passengers per bus.
D) the number of buses required for 25,000 passengers.
E) None of the above
11) The standard quota of Route $A$ (to 2 decimal places) is
11)
A) 7.14 .
B) 12.78 .
C) 6.39 .
D) 63.88 .
E) None of the above
12) In process of applying Hamilton's method, the route receiving the "extra" bus is
12)
A) Route B.
B) Route C.
C) Route A.
D) Route D.
E) None of the above
13) Find the apportionment of the buses among the routes using Hamilton's method.
A) Route A: 7; Route B: 18; Route C: 9; Route D: 16
B) Route A: 7; Route B: 18; Route C: 10; Route D: 16
C) Route A: 6; Route B: 18; Route C: 9; Route D: 17
D) Route A: 7; Route B: 17; Route C: 9; Route D: 17
E) None of the above
14) Find the apportionment of the buses among the routes using Jefferson's method.
A) Route A: 7; Route B: 18; Route C: 10; Route D: 16
B) Route A: 6; Route B: 18; Route C: 9; Route D: 17
C) Route A: 7; Route B: 17; Route C: 9; Route D: 17
D) Route A: 7; Route B: 18; Route C: 9; Route D: 16
E) None of the above
15) Find the apportionment of the buses among the routes using Adams' method.
A) Route A: 7; Route B: 17; Route C: 9; Route D: 17
B) Route A: 6; Route B: 18; Route C: 9; Route D: 17
C) Route A: 7; Route B: 18; Route C: 9; Route D: 16
D) Route A: 7; Route B: 18; Route C: 10; Route D: 16
E) None of the above
16) Find the apportionment of the buses among the routes using Webster's method.
15)
14)
13) $\qquad$
$\qquad$
 $\qquad$
16) $\qquad$
A) Route A: 7; Route B: 18; Route C: 10; Route D: 16
B) Route A: 7; Route B: 17; Route C: 9; Route D: 17
C) Route A: 7; Route B: 18; Route C: 9; Route D: 16
D) Route A: 6; Route B: 18; Route C: 9; Route D: 17
E) None of the above

A country has four states. Suppose the population of State 1 is $P_{1}$, the population of State 2 is $P_{2}$, the population of State 3 is $P_{3}$, and the population of State 4 is $P_{4}$. Suppose also that the total number of seats in the legislature is $M$ and the standard divisor is $D$.
17) The value of $D$ is
17)
A) $P_{1}+P_{2}+P_{3}+P_{4}$.
B) $\frac{P_{1} \times P_{2} \times P_{3} \times P_{4}}{M}$.
C) $\frac{P_{1}+P_{2}+P_{3}+P_{4}}{M}$.
D) $\frac{M}{P_{1}+P_{2}+P_{3}+P_{4}}$.
E) None of the above
18) If $\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{q}_{3}$, and $\mathrm{q}_{4}$ are the respective standard quotas for the four states, then
18) $\mathrm{q} 1+\mathrm{q} 2+\mathrm{q} 3+\mathrm{q} 4$ equals
A) the number of seats in the legislature M .
B) 0 .
C) the total population $\mathrm{P}_{1}+\mathrm{P}_{2}+\mathrm{P}_{3}+\mathrm{P}_{4}$.
D) the standard divisor D .
E) None of the above
19) If $J$ is the modified divisor used for Jefferson's method, then
19)
A) J can be less than, equal to, or greater than D.
B) $J$ is always greater than or equal to $D$.
C) $J$ is always equal to $D$.
D) J is always less than or equal to D .
E) None of the above

## Solve the problem.

20) Which of the following apportionment methods does not violate the quota rule?
21) 

A) Adams' method
B) Hamilton's method
C) Jefferson's method
D) Webster's method
E) None of the above
21) Which of the following apportionment methods can produce the Population paradox?
21) $\qquad$
A) Adams' method
B) Jefferson's method
C) Webster's method
D) Hamilton's method
E) None of the above
22) In a certain apportionment problem, State $X$ has a standard quota of 48.9. The final apportionment to State X is 50 seats. This is called
A) an upper-quota violation.
B) the population paradox.
C) the Alabama paradox.
D) a lower-quota violation.
E) None of the above
23) A father wishes to distribute 16 pieces of candy among his 3 children (Abe, Betty, and Cindy) based on the number of hours each child spends doing chores around the house. Using a certain apportionment method, he has determined that Abe is to get 9 pieces of candy, Betty is to get 4 pieces, and Cindy is to get 3 pieces. However, just before he hands out the candy, he discovers that he has 17 pieces (not 16) of candy. When he apportions the 17 pieces of candy using the same apportionment method, Abe ends up with 10 pieces, Betty with 5 pieces, and Cindy with 2 pieces. This is an example of
A) the new states paradox.
B) the Alabama paradox.
C) a violation of the quota rule.
D) the population paradox.
E) None of the above

## The figure below is a square $A B C D$ with center O . ( $\mathrm{M}, \mathrm{N}, \mathrm{P}$, and Q are the midpoints of the sides.)


24) Which of the following reflections is not a symmetry of the square?
A) the reflection with axis the line passing through $A$ and $C$
B) the reflection with axis the line passing through $P$ and $Q$
C) the reflection with axis the line passing through $A$ and $B$
D) the reflection with axis the line passing through M and N
E) All of the above are symmetries of the square.
25) Which of the following rotations is a symmetry of the square?
23) $\qquad$
26) Which of the following translations is a symmetry of the square?
A) a translation that sends $A$ to $C$
B) a translation that sends A to B
C) a translation that sends $P$ to $Q$
D) a translation that sends A to O
E) None of the above
27) The image of $A$ under the reflection with axis the line passing through $M$ and $P$ is
27)
A) C.
B) D .
C) B.
D) O .
E) None of the above
28) The image of A under a $90^{\circ}$ clockwise rotation with center O is
28)
A) C.
B) B.
C) $D$.
D) M .
E) None of the above
29) A translation sends the point $A$ to the point $Q$. The image of $P$ under this translation is
A) O .
B) $B$.
C) C .
D) N .
E) None of the above
30) A glide reflection sends the point $A$ to the point $Q$ and the point $P$ to the point $C$. The image of $B$
30) under this glide reflection is
A) P.
B) D .
C) A.
D) N .
E) None of the above
31) A glide reflection sends the point $A$ to the point $Q$ and the point $P$ to the point $C$. The axis of this
31) glide reflection is a line passing through the points
A) $P$ and $Q$.
B) $M$ and $N$.
C) A and B.
D) A and $Q$.
E) None of the above

## Solve the problem.

32) A $7216^{\circ}$ clockwise rotation is equivalent to
A) a $344^{\circ}$ counterclockwise rotation.
B) a $376^{\circ}$ clockwise rotation.
C) a $16^{\circ}$ clockwise rotation.
D) All of the above
E) None of the above
33) A glide reflection having axis of reflection as shown below sends point $P$ to point $Q$. The image of point R under this same glide reflection is

A) A.
B) B.
C) C.
D) D.
E) None of the above
34) The letter $C$ has a symmetry type
A) $Z_{2}$.
B) $\mathrm{D}_{2}$.
C) $\mathrm{D}_{1}$.
D) $\mathrm{Z}_{1}$.
E) None of the above
35) The letter $Q$ has a symmetry type
$\qquad$
A) $Z_{2}$.
B) $\mathrm{D}_{1}$.
C) $Z_{1}$.
D) $\mathrm{D}_{2}$.
E) None of the above
36) The letter $Z$ has a symmetry type
A) $\mathrm{D}_{2}$.
B) $\mathrm{Z}_{1}$.
C) $\mathrm{D}_{1}$.
D) $Z_{2}$.
E) None of the above
37) If an object has a $30^{\circ}$ clockwise rotation as one of its symmetries, then it must also have as a symmetry
A) a $90^{\circ}$ clockwise rotation.
B) a $45^{\circ}$ clockwise rotation.
C) a translation.
D) a reflection.
E) None of the above
38) The complete symmetries of the border pattern $\ldots \mathrm{Z} Z \mathrm{Z} \mathrm{Z} \mathrm{Z} \mathrm{Z} \ldots$ are the identity and
A) translations and $180^{\circ}$ rotations only.
B) translations and $45^{\circ}$ rotations only.
C) translations and horizontal reflections only.
D) translations and vertical reflections only.
E) None of the above
39) The complete symmetries of the border pattern . . . pbqd pbqd pbqd... are the identity and
40) 

A) translations and glide reflections only.
B) translations and $180^{\circ}$ rotations only.
C) translations, glide reflections, and $180^{\circ}$ rotations only.
D) translations only.
E) None of the above

Refer to the figures and recursive rules below to answer the following question(s).


Figure 1


Figure 2


Figure 3


Figure 4

Rule A:

- Start with a solid black equilateral triangle.
- Whenever you see an edge $\frac{\text { white }}{\text { black }}$ replace it with _ .


## Rule B:

- Start with a solid black triangle.
- Whenever you see a replace it with a

Rule C:

- Start with a solid black square.
- Whenever you see an edge $\frac{\text { white }}{\text { black }}$ replace it with $\square$.

Rule D:

- Start with a solid black square.
- Whenever you see a square, subdivide the square into nine equal subsquares and remove the central subsquare.
Rule E:
- Start with a solid black equilateral triangle.
- Whenever you see an edge $\frac{\text { white }}{\text { black }}$ replace it with $\frac{\text { white }}{\text { black black }}$.

40) Which of the figures above approximates the result of recursively applying Rule A infinitely many times?
A) Figure 1
B) Figure 2
C) Figure 3
D) Figure 4
E) None of the above
41) Which of the figures above approximates the result of recursively applying Rule B infinitely $\qquad$ many times?
A) Figure 1
B) Figure 2
C) Figure 3
D) Figure 4
E) None of the above
42) Which of the figures above approximates the result of recursively applying Rule C infinitely
43) many times?
A) Figure 1
B) Figure 2
C) Figure 3
D) Figure 4
E) None of the above
44) Which of the figures above approximates the result of recursively applying Rule D infinitely many times?
A) Figure 1
B) Figure 2
C) Figure 3
D) Figure 4
E) None of the above
45) Which of the figures above approximates the result of recursively applying Rule E infinitely
46) $\qquad$ many times?
A) Figure 1
B) Figure 2
C) Figure 3
D) Figure 4
E) None of the above

## Solve the problem.

45) If the area of the starting triangle in the construction of the Koch snowflake is 5 , then the area of the Koch snowflake is
A) 8 .
B) 10 .
C) infinite.
D) 0 .
E) None of the above
46) Suppose that the perimeter of the starting triangle in the construction of the Koch snowflake is 5.
47) Then the length of the boundary of the Koch snowflake is
A) 0 .
B) infinite.
C) 10 .
D) 8 .
E) None of the above

The following question(s) refer to a fractal defined by the recursive procedure :

- Start with a line segment of length 5.
- Step 1: Replace the line segment with _- (see figure below).
- Step 2: Replace each line segment in the previous figure with _re (see figure below).
- Step 3, 4, 5, etc.: Replace each line segment in the prewious figure with $\qquad$ .


47) What is the length of the figure at step 1 of the construction?
A) 5
B) $3+2 \sqrt{2}$
C) $5 \sqrt{2}$
D) $7+2 \sqrt{2}$
E) None of the above
48) How many square units of area are added above the original horizontal line segment at step 1 of
49) the construction?
A) 4
B) 1
C) 3
D) 2
E) None of the above
50) How many line segments appear in step 2 of the construction? $\qquad$
A) 52
B) 5
C) 25
D) $5 \times 2$
E) None of the above
51) How many line segments appear in step 4 of the construction?
A) $5 \times 4$
B) 45
C) 5
D) 54
E) None of the above
52) What is the length of the leftmost line segment in step 3 of the construction?
A) $\left(\frac{1}{5}\right)^{3}$
B) $\left(\frac{1}{5}\right)^{2}$
C) $\frac{1}{3}$
D) $\left(\frac{1}{3}\right)^{5}$
E) None of the above

## Solve the problem.

52) Of the following objects in nature, which one could never have symmetry of scale?
53) 

A) a mountain
B) a coastline
C) a soap bubble
D) a cloud
E) All of the above could have symmetry of scale.

Answer Key
Testname: 101PRACMT2

1) D 50) D
2) $D$
3) $A$
4) B
5) C
6) $C$
7) $A$
8) $B$
9) A
10) C
11) C
12) C
13) C
14) A
15) B
16) C
17) D
18) C
19) A
20) $D$
21) B
22) $D$
23) $A$
24) B
25) C
26) B
27) E
28) D
29) B
30) C
31) A
32) B
33) $D$
34) D
35) C
36) C
37) D
38) A
39) A
40) B
41) C
42) A
43) B
44) D
45) E
46) A
47) B
48) B
49) $D$
50) A
