Math 115 Midterm 1 Solutions

1. A Gallup poll asked, "Do you think the U.S. should take the leading role in world affairs, take a major role but not the leading role, take a minor role, or take no role at all in world affairs?" Gallup's report said, "Results are based on telephone interviews with 1,002 national adults, aged 18 and older, conducted Feb. 9-12, 2004."

(a) What is the population for this sample survey? What was the sample size?

(b) Gallup notes that the order of the four possible responses was rotated when the question was read over the phone. Why was this done?

(This is problem 3.37 from your textbook.) (a) Population is adult residents of the U.S. Sample size is 1002. (b) Because the order might affect people's answers (they might be more likely to pick the first answer, for example). Rotating the order should cancel this out.

2. The following are the final exam scores of the 13 students in an Applied Poetry course:

116 118 121 123 124 124 124 132 133 144 156 163 179

(a) Draw a stemplot of the data.

(b) Give the 5-number summary.

(c) Are there any suspected outliers according to the 1.5 IQR criterion?

(d) Based on the shape of the distribution, is the mean or the median a better measure of the center of the data? Explain.

(a)

stem	leaf
11	68
12	13444
13	23
14	4
15	6
16	3
17	9

(b) The five-number summary is (min, Q1, median, Q3, max), so here it's (116, 122, 124, 150, 179).

(c) IQR = Q3 - Q1 = 150 - 122 = 28. 1.5IQR = 42. Q1 - 1.5IQR = 122 - 42 = 80, and Q3 + 1.5IQR = 150 + 42 = 192. All of our data are within the range (80, 192), so there are no suspected outliers.

(d) Because the distribution is skewed to the right, the mean will be large, so the median is a better measure of center.

3. You own a small statistical consulting business. The salaries of your three employees are $32,000, $36,000, and $49,000.

(a) Find the mean of the salaries.

(b) Find the standard deviation of the salaries.

(c) Business has been so good this year that for next year, you've decided to give each of your employees an 8% raise and a $2,000 bonus. What are the mean and standard deviation of the salaries next year? (HINT: Giving someone an 8% raise is the same as multiplying her salary by 1.08.)

(a) mean=(32000+36000+49000)/3=39000

(b) s.d. $= \sqrt{\dfrac{(32000-39000)^2+(36000-39000)^2+(49000-39000)^2}{3-1}} = 8888.2.$

(c) Recall that adding a constant (2000) increases the mean by that constant and does nothing to the spread, while multiplying by a constant (1.08) multiplies the mean by that constant and the s.d. by the absolute value of the constant. So the new mean is $1.08 \cdot 39000 + 2000 = 44120$**, and the new s.d. is** $1.08 \cdot 8888.2 = 9599.3$**.**

4. The scores of a reference population on the Wechsler Intelligence Scale for Children (WISC) are normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$.

(a) What proportion of the population has a WISC score between 110 and 130?

(b) What is the WISC score that only 10% of the population exceeds?

(a) Convert to standard normal by finding Z-scores. The Z-score for 110 is $\frac{110-100}{15} \approx 0.67$**, and the Z-score for 130 is** $\frac{130-100}{15} = 2$**. So the probability that the WISC score is between 110 and 130 is** $P(0.67 < Z < 2) = .9772 - .7486 = .2286$ **(from Table A).**

(b) In other words, what is the WISC score that 90% of the population is below? Table A tells us that 90% (actually, 89.97%) of Z-scores are below 1.28. (That is, 90% of normally distributed data is less than 1.28 s.d.'s above the mean.) The corresponding WISC score is $100 + 1.28 \cdot 15 = 119.2$**.**

5. Students in a statistics class at Angus Scott College in Scotland measured the length of their forearms and their heights in inches. Suppose that a tailor wants to predict the length of a student's forearm when he knows the student's height. Fathom gives the following output:

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Equation of least-squares regression line: FOREARM_LENGTH = 0.21 HEIGHT - 3.42
r = .81
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(a) What forearm length would the tailor predict for a student who is 73 inches tall?

(b) Interpret the slope 0.21 in this context.

(c) If the average height is 69 inches, what is the average forearm length?

(d) What fraction of the variation in forearm length is explained by the linear regression line?

(a) Just plug in: length = $0.21 \cdot 73 - 3.42 = 11.91$ **inches.**

(b) It says that an increase of 1 inch in height is associated with an increase of 0.21 inches in forearm length.

(c) Recall that the point $(\bar{x}, \bar{y})$ **is always on the regression line, so if we plug in the mean height 69, we'll get the mean forearm length:** $0.21 \cdot 69 - 3.42 = 11.07$ **inches.**

(d) This is $r^2 = (.81)^2 = .6561$**.**