Math 115 Midterm #2 Solutions

1. (30 pts) Label each of the following questions with the letter of the technique that you would use to answer it. (Some letters may be used more than once; some may not be used at all.)

(i) How much longer do women live than men? _____

(ii) Are basketball players taller than soccer players? _____

(iii) Do college students have a higher GPA in their first year than in their second? _____

(iv) How much does the average squirrel weigh? _____

(A) One-sample t-test; find a confidence interval.
(B) One-sample t-test; hypothesis test.
(C) Matched pairs one-sample t-test; find a confidence interval.
(D) Matched pairs one-sample t-test; hypothesis test.
(E) Two-sample t-test; find a confidence interval.
(F) Two-sample t-test; hypothesis test.

i:E, ii:F, iii:D, iv:A

2. (30 pts) Lake Springfield is contaminated with toxic waste, so the fish are mutated and have between 0 and 3 eyes. Let the random variable X be the number of eyes on a randomly selected fish.

X	0	1	2	3
Prob	0.2	0.1	???	0.3

(a) What is the missing value **???** in the probability distribution table above?
(b) What is the average number of eyes? (I.e., what is μ_X?)
(c) What is the probability that a given fish has at least two eyes?
(d) You catch four different fish. Assuming that their mutations are independent, what is the probability that at least two of them have *exactly* one eye?

(a) The probabilities have to sum to 1, so it's 0.4.
(b) $\mu_{\mathbf{x}} = \mathbf{0(0.2) + 1(0.1) + 2(0.4) + 3(0.3) = 1.8}$
(c) $\mathbf{P(X = 2) + P(X = 3) = 0.4 + 0.3 = 0.7}$.
(d) The probability that a given fish has exactly one eye is 0.1. Thus the number Y **of the four fish who have exactly one eye has the Binomial(4, 0.1) distribution. The probability that at least two have exactly one eye is** $\mathbf{P(Y = 2) + P(Y = 3) + P(Y = 4)}$**, which Table C tells us is** $.0486 + .0036 + .0001 = .0523$**.**

3. (10 pts) A one-sided test of the null hypothesis $\mu = 50$ versus the alternative $\mu = 70$ has power equal to 0.5. Will the power for the alternative $\mu = 80$ be higher or lower than 0.5? Explain your answer.

(This is 6.93 from your homework.) The power will be higher, because larger differences are easier to detect.

4. (30 pts) Do drivers talking on cellphones have longer reaction times than drivers listening to the radio? Scientists at the University of Utah tested this, by randomly assigning subjects to a cellphone group or a radio group, and then measuring how long it took them to hit the brakes after seeing a red light (in a driving simulator, of course, not on the road). Their data are below (response times are in milliseconds).

Group	n	Sample Average Response Time	Sample Std. Dev.
Cellphone users	32	585.2	89.6
Radio listeners	32	533.7	65.3

(a) Define parameters and carry out a test to see if there is significant evidence that cellphone users have a longer mean response time than radio listeners. Report your P-value and conclusion.

(b) Which of the following statements gives the best interpretation of the P-value in part (a)? Choose one (i-v):_____

(i) It is the probability that the mean response time is shorter for cellphone users than for radio listeners.

(ii) It is the probability that the mean response time is longer for cellphone users than for radio listeners.

(iii) It is the probability of seeing a sample with average cellphone user response time that is 51.5 milliseconds or more longer than the sample average radio listener response time, if the means are the same.

(iv) It is the probability of seeing a sample with average cellphone user response time that is 51.5 milliseconds or less longer than the sample average radio listener response time, if the means are the same.

(v) It is the probability that the mean response time is the same for both groups.

(a) Use a 2-sample t-test. Let X_1 be the response time of a random cellphone user (with mean μ_1), and X_2 the response time of a random radio listener (with mean μ_2). Then $H_0 : \mu_1 = \mu_2$, and $H_a : \mu_1 > \mu_2$ (one-sided: we're asking whether the cellphone user response time is *longer*, not just different). The t-statistic is $\frac{(585.2-533.7)-0}{\sqrt{\frac{89.6^2}{32}+\frac{65.3^2}{32}}} \approx \frac{51.5}{19.60} \approx 2.63$. We approximate our degrees of freedom by $32 - 1 = 31$, so we use the row for df=30 on Table D to conclude that our P-value is between 0.005 and 0.01. Therefore we can reject H_0 at level $\alpha = 0.01$, and we conclude that there is strong evidence that drivers talking on cellphones have longer reaction times than drivers listening to the radio.

(b) (iii)