Math 115 Midterm #2 Solutions

**1.** (30 pts) Customers at Mr. Gatti's Pizza order pizzas with different numbers of toppings. Let the random variable $X$ be the number of toppings on the pizza ordered by a randomly selected customer.

| **X** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Prob** | 0.15 | 0.30 | 0.35 | 0.15 | **???** |

(a) What is the missing value **???** in the probability distribution table above?
(b) What is the average number of toppings ordered? (I.e., what is $\mu_X$?)
(c) What is the probability that a given customer orders at least two toppings?
(d) Five different customers order pizzas. Assuming that their orders are independent, what is the probability that at least two of them order *exactly* three toppings?

**(a) The probabilities have to sum to 1, so it's 0.05.**
**(b)** $\mu_{\mathbf{x}} = \mathbf{0(0.15) + 1(0.30) + 2(0.35) + 3(0.15) + 4(0.05) = 1.65}$
**(c)** $\mathbf{P(X = 2) + P(X = 3) + P(X = 4) = .35 + .15 + .5 = .55}$
**(d) The probability that a given customer orders exactly three toppings is 0.15. Thus the number** $Y$ **of the five customers who order exactly three toppings has the Binomial(5, 0.15) distribution. The probability that at least two order exactly three is** $\mathbf{P(Y = 2) + P(Y = 3) + P(Y = 4) + P(Y = 5)}$**, which Table C tells us is** $.1382 + .0244 + .0022 + .0001 = .1649$**.**

**2.** (30 pts) At Angus Scot College in Decatur, Scotland, each semester students in the intro stats class measure the lengths of their kilts as part of a class survey. There could be a systematic difference between the kilts worn on the first days of the fall semester and those worn on the first day of the spring semester (due to the weather, for example). Below is a table with the sample average and standard deviation of kilt lengths in inches for surveys done this fall and last spring:

| **Semester** | **n** | **Mean** | **Std. Dev.** |
|---|---|---|---|
| Fall | 98 | 21.44 | 2.42 |
| Spring | 101 | 22.40 | 2.18 |

(a) Consider these to be simple random samples of 2005-2006 fall and spring semester Angus students (and their kilts). Define parameters and carry out a test to see if there is significant evidence for a difference in mean kilt lengths for the fall and spring semesters. Report your P-value and conclusion.
(b) Which of the following statements gives the best interpretation of the P-value in part (a)? Choose one (i-v):_____
  (i) It is the probability that the mean kilt length is the same for both semesters.
  (ii) It is the probability that the mean kilt length is different for fall and spring.
  (iii) It is the probability of seeing averages that differ by exactly 0.96 inches if the means are the same.
  (iv) It is the probability of seeing averages that differ by 0.96 inches or more if the means are the same.
  (v) It is the probability of seeing averages that differ by 0.96 inches or less if the means are the same.
(c) Construct a 99% confidence interval for the difference in the means.

**(a) If we had measured the same students in each semester, we could have done a matched-pairs one-sample** $t$**-test. Since we didn't, we'll use a two-sample** $t$**-test. Let** $X_1$ **be the length of a random kilt in the fall semester (with mean** $\mu_1$**), and** $X_2$ **the length of a random kilt in the spring semester (with mean** $\mu_2$**). Then** $H_0$**:** $\mu_1 = \mu_2$**,**

**and $H_a$: $\mu_1 \neq \mu_2$. Our test statistic is $t = \frac{22.4-21.44}{\sqrt{\frac{2.18^2}{101}+\frac{2.42^2}{98}}} \approx \frac{.96}{.3268} \approx 2.94$. Use df$= 98-1 = 97$. We skip down to df$= 80$ in Table D (we could probably also skip up to 100) to get a one-sided P-value of between .0025 and .001, so our two-sided P-value is between .005 and .002. Therefore we can reject $H_0$ at level $\alpha = .005$, and we conclude that there is strong evidence that kilt lengths are different in the fall and spring.**

**(b) (iv)**

**(c) In part a, we found the standard error to be .3268. The $t(80)$ 99% critical value is 2.639, so the CI is $.96 \pm (2.639)(.3268) = (0.097, 1.822)$. If we use 100 for df, the critical value is 2.626, and the CI is $96 \pm (2.626)(.3268) = (0.102, 1.818)$.**

**3.** (30 pts) Label each of the following questions with the letter of the technique that you would use to answer it. (Some letters may be used more than once; some may not be used at all.)

(i) How much more money do lawyers make than math professors? _____

(ii) Are pears from Publix heavier than pears from Kroger? _____

(iii) Do math 115 students do better on the second midterm than on the first? _____

(iv) How many chocolate chips are there in a Chips Ahoy cookie? _____

(A) One-sample $t$-test; find a confidence interval.

(B) One-sample $t$-test; hypothesis test.

(C) Matched pairs one-sample $t$-test; find a confidence interval.

(D) Matched pairs one-sample $t$-test; hypothesis test.

(E) Two-sample $t$-test; find a confidence interval.

(F) Two-sample $t$-test; hypothesis test.

**i:E, ii:F, iii:D, iv:A**

**4.** (10 pts) A local television station announces a question for a call-in opinion poll on the six o'clock news and then gives the response on the eleven o'clock news. Today's question concerns a proposed increase in funds for student loans. Of the 2372 calls received, 1921 oppose the increase. The station, following standard statistical practice, makes a confidence statement: "81% of the Channel 13 Pulse Poll sample oppose the increase. We can be 95% confident that the proportion of all viewers who oppose the increase is within 1.6% of the sample result." Is the station's conclusion justified? Explain your answer.

**(This is 6.77 from the text.) The conclusion is not justified, because of response bias in the survey: the sample isn't random, because it consists only of people who cared enough to call in. We really can't conclude anything from these data.**