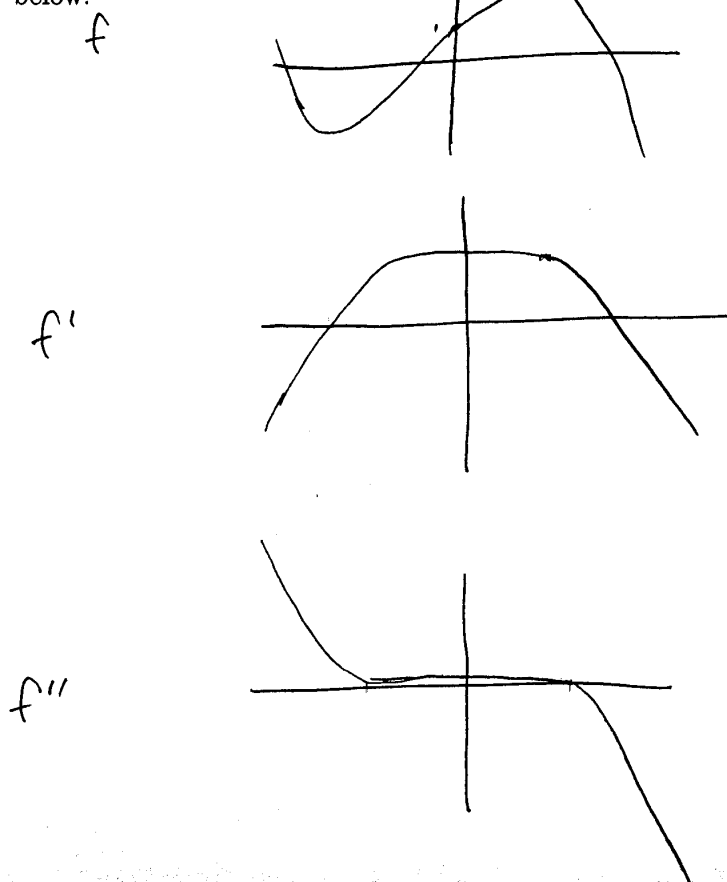


1. (20 pts) A parachutist jumps out of a plane flying at 10,000 feet. Let $f(t)$ be the parachutist's height above ground, in feet, t seconds after she jumps out. Label each expression on the left with the letter, or letters, of the matching physical quantity on the right. If it doesn't match any of the physical quantities, label it "NONE." (Each expression may match more than one physical quantity, or it may match none. Similarly, each physical quantity may match more than one expression, or it may match none.)

- | | |
|---|--|
| (i) $f(2)$ <i>A</i> | (a) Height after 2 seconds |
| (ii) $f''(2)$ <i>F</i> | (b) Time when parachutist hits the ground |
| (iii) $f(2) - f(0)$ <i>NONE</i> | (c) Instantaneous velocity after 2 seconds |
| (iv) $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$ <i>C</i> | (d) Average height over first 2 seconds |
| (v) $\frac{f(2) - f(0)}{2}$ <i>G</i> | (e) Instantaneous acceleration after t seconds |
| (vi) $\lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h}$ <i>NONE</i> | (f) Instantaneous acceleration after 2 seconds |
| (vii) $10,000 - f(2)$ <i>NONE</i> | (g) Average velocity over first 2 seconds |
| (viii) $f'(2)$ <i>C</i> | (h) Rate of change of acceleration after 2 seconds |

2. (20 pts) I'm thinking of a certain function $f(x)$. You know that $f(0) = 1$, and the graph of $f'(x)$ is given below. (That's $f'(x)$, **not** $f(x)$.) Sketch the graphs of $f(x)$ and $f''(x)$ on the axes below.



3. (20 pts) Let w be the weight, in pounds, of a refrigerated turkey. Let $f(w)$ be the time, in ~~minutes~~ ^{hours}, required to cook the turkey completely in a 400° oven.

(a) Explain, in words, the meaning of the following:

(i) $f(20) = 6$

It takes 6 hrs. to cook a 20 lb. turkey.

(ii) $f^{-1}(3)$ The weight of a turkey that takes 3 hours to cook.

(iii) $f'(10)$ Roughly, the number of extra hours it would take to cook an 11 lb. turkey instead of a 10 lb. turkey.

(b) Do you expect $f'(12)$ to be positive or negative? Why?

Positive, because bigger turkeys take longer to cook.

(c) True or false: Turkey is delicious. True

4. (20 pts) My arch-enemy, Sideshow Bob, snuck a big bowl of potato salad into my refrigerator. Now mold is growing on it. I've computed that the amount $M(t)$ of mold (measured in pounds) t hours after midnight is given by

$$M(t) = 0.2e^{1.1t}$$

(a) How long does it take the amount of mold to double?

Solve for t : $M(t) = 2M(0)$, $0.2e^{1.1t} = 2 \cdot 0.2e^{1.1 \cdot 0}$, $0.2e^{1.1t} = 0.4$,
 $e^{1.1t} = 2$, $1.1t = \ln 2$, $t = \frac{\ln 2}{1.1}$ hours

(b) How fast is the amount of mold growing at 6 am? (Give units.)

$M'(t) = (0.2e^{1.1t})' = 0.2(e^{1.1t})' = 0.2(1.1e^{1.1t}) = 0.22e^{1.1t}$
 So $M'(2) = \cancel{0.22} 0.22e^{2.2}$ lbs/hr

5. (20 pts) The following table gives US population (in millions) during the nineteenth century.

Year	1800	1820	1840	1860	1880	1900
Population (millions)	5.3	9.6	17.1	31.4	50.2	76.0

(a) Roughly how fast is the population growing in 1850? (Give units.)

$$\begin{aligned} &\approx \text{average growth rate between 1840 \& 1860} = \frac{P(1860) - P(1840)}{1860 - 1840} \\ &= \frac{31.4 - 17.1}{20} = \frac{14.3}{20} = .715 \text{ million/yr} = 715,000 \text{ people/yr} \end{aligned}$$

(b) If $P(t)$ is the population in year t , what can you say about the second derivative of P ?

Popn. grows ~ 4 mil 5/w 1800 & 1820
 7 mil " 1820 & 1840
 14 mil " 1840 & 1860
 19 mil " 1860 & 1880
 26 mil " 1880 & 1900

Rate of popn. growth is increasing, so the second derivative is positive.

EXTRA CREDIT Who's better, Newton or Leibniz?