MATH 118 PROJECT #3 (STOLEN FROM DR. KOCH)
Due Wednesday, November 28.

Work in groups of three to five people. Turn in a report of your results (with complete sentences, please, and graphs and figures as appropriate) and your pizza box in class on Wednesday, November 28 – one report per group, pledged by all of the group members. Think of a classmate who hasn't thought about these questions as the target audience for your report. *You may not discuss your work with members of any other group.*

You've decided to open your own pizza place, complete with hilarious name and logo. Now you need boxes to put your pizzas in. The purpose of this project is to design and build a pizza box. Each box is made from a piece of cardboard that is $8\frac{1}{2}$ inches by 11 inches. The design will be as follows: six squares of width x are to be cut from the cardboard, which will then be folded into a box of height x, as shown below. Your job is to determine the value of x that will maximize the volume of the box.
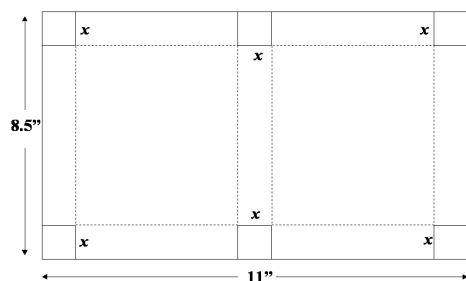
The Objective Function.
(1) Find all of the dimensions of the box in terms of x.
(2) Construct a function $V(x)$ which gives the volume of the box.
(3) What is the domain of $V(x)$? Take into consideration the size of the piece of cardboard.

The Solution.
(4) Use your graphing calculator to sketch a graph of $V(x)$ on its domain. Provide a sketch with your write-up, putting units on your axes.
(5) Based on #4, what do you estimate for the optimal value for x?
(6) Now, using calculus, find the x value a that maximizes $V(x)$.
(7) What is the largest volume you can obtain by making a box out of your piece of paper?

The Construction. Take an $8\frac{1}{2} \times 11$ piece of cardboard and build the box that maximizes volume. Make sure to add the name and logo of your restaurant.

Uh... Aren't Pizzas Round? Okay, you've built the largest pizza box you possibly could. If your pizza is rectangular, this box maximizes the size of the pizza that will fit inside. Suppose you want – oh I don't know – a *circular* pizza. Actually, your pizza will have height as well, so it's technically a cylinder.
(8) Using your optimum choice of x above, what is the volume of the largest cylindrical (circular, with height) pizza that can fit in your box? Don't worry about things like "the height of the pizza cannot be exactly equal to the height of the box because all the cheese will stick to the top" – assume your pizza can be as tall as the box allows.
(9) Construct a new $V(x)$ that gives the volume of the circular pizza inside a box constructed by cutting out the six squares from the $8\frac{1}{2} \times 11$ sheet of cardboard.
(10) Find the choice of x that maximizes the size of the cylindrical pizza you can fit inside. You do not need to build it.
(11) What are the dimensions of the largest cylindrical pizza? What is its volume?