Math 118 Practice Final Exam

1. When I first moved to Atlanta, it rained every day for two weeks. One day, I decided to measure how much it was raining. Here is a selection of the data that I gathered (t is the number of hours after noon, and $f(t)$ is the number of inches of rain that had fallen since noon).

t	3.0	3.2	3.4	3.6	3.8
$f(t)$	0.82	0.95	1.05	1.10	1.32

(a) What was the average rate at which the rain was falling between $t = 3.0$ and $t = 3.8$?
(b) Estimate $f'(3.0)$ (including units).

2. Compute the following derivatives:

(a) $\dfrac{d}{dx}(\cos(3x^2+1))$
(b) $\dfrac{d}{dx}\left(\dfrac{4^x}{7-x}\right)$
(c) $\dfrac{d}{dt}(5+3t^2e^{-t})$

3. Compute the following integrals:

(a) $\displaystyle\int_{-\pi}^{0} \sin(t)\, dt$
(b) $\displaystyle\int \frac{2}{t}\, dt$
(c) $\displaystyle\int_{1}^{3} \frac{2}{x^3}\, dx$
(d) $\displaystyle\int_{-2}^{2} |x|\, dx$

4. Use linear approximation to estimate $\ln(0.9)$. (Hint: $a = 1$ may be a good choice.)

5. A truck full of fashionable shoes leaves the factory at noon and travels along a straight road. The truck's velocity (in miles per hour) is given by $v(t) = 30t - 3t^2$, where t is the number of hours since noon. If you live 10 miles down the road from the factory, will the truck have passed your house by 1:00 pm?

6. You've just been hired as the new president of Delta Airlines. Your underlings tell you that the regular air fare between Atlanta and Austin is \$500. Delta flies 747s (which can hold up to 380 people) on this route, and averages 300 passengers. Market research indicates that each \$1 fare reduction would attract, on average, 1 more passenger for each flight (and conversely: each \$1 fare increase would reduce the average number of passengers per flight by 1). How should you set the fare to maximize Delta's revenue?

7. Let $f(t)$ be the total number of gallons of delicious chocolate milk that Hilda has consumed by age t (years). Interpret the following in practical terms. (HINT: units!)

(a) $f(14) = 400$
(b) $f^{-1}(50) = 6$
(c) $f'(12) = 50$
(d) $(f^{-1})'(450) = 1/70$

8. I'm filling a spherical balloon with water. The volume V of the balloon depends on the radius r: $V = \frac{4}{3}\pi r^3$. After t seconds, the radius is $r(t)$ cm.
(a) Explain in words the meanings of the following derivatives (give units): dV/dr, dr/dt, dV/dt.
(b) The radius is increasing at 2 cm/sec. At what rate (cm^3/sec) is the volume increasing when the radius is 10 cm?