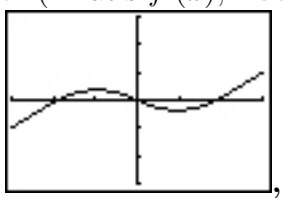
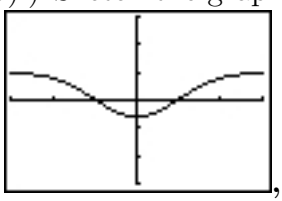
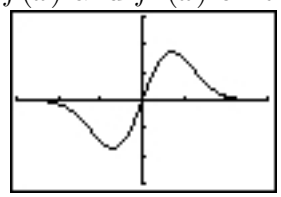
Math 118 February 18, 2008 Name:_____

MIDTERM #1 solutions

1. Find the equation of the line tangent to the graph of $f(x) = 4x + \dfrac{1}{x}$ when $x = -1$.

The slope of the tangent line is the derivative, $f'(-1)$. Since $\frac{d}{dx}(4x+1/x) = 4-1/x^2$, $f'(-1)$=2-1/1=3. When x=-1, f(x)=-4-1=-5, so the tangent line passes through the point (-1,-5). So the tangent line is y-(-5)=3(x-(-1)), or y+5=3x+3, or y=3x-2.

2. You know two things about the function $f(x)$: $f(0) = 0$, and the graph of $f'(x)$ is as shown below. (That's $f'(x)$, **not** $f(x)$!) Sketch the graphs of $f(x)$ and $f''(x)$ on the axes below.

f:  , **f':**  , **f":** 

3. It's well known that the number of whiskers on a yeti's head is proportional to the cube of the yeti's height. If a 2-meter-tall yeti has 1000 whiskers, how tall is a yeti that has 700 whiskers?

We have that $w = kh^3$, so 1000=k*8, or k=125. So 700=125h^3, or $h = \sqrt[3]{700/125}$ ≈1.776 meters.

4. Let $f(t)$ be the average height of a t-year-old female in the United States in the years 1999-2002. We have the following data (from the CDC):

t	$f(t)$
11	59.6
12	61.4
13	62.6
14	63.7

(a) Estimate $f'(14)$.

(b) Use your answer to (a) to estimate the average height of a 15-year-old female.

(a) We'll use the average rate of change from year 13 to year 14 for our estimate. The average rate is (63.7 - 62.6)/(14 - 13) = 1.1 inches/year. (b) 63.7 + 1.1*1 = 64.8 inches. (According to the CDC, the average height is actually 63.8. What might account for the difference between our estimate and the real answer?)

5. Becky's heart rate, H (measured in beats/minute), is a function of the amount of coffee she's drunk, c (measured in liters), so $H = f(c)$.

(a) Explain, in words, the meaning of the following expressions or equations. (Your explanations should be understandable by someone who hasn't taken calculus. Units may be helpful.)

(i) $f(2) = 120$

(ii) $f^{-1}(90)$

(iii) $f'(2) = 30$

(b) Use the information above to estimate her heart rate when she's drunk 1.5 liters of coffee.

(a)(i) If she's drunk 2L, her heart rate is 120. (ii) The amount that she must have drunk if her heart rate is 90. (iii) When she's drunk 2L, her heart rate will go up by about 30 beats/minute for every additional liter she drinks. (b) This is 0.5L less than 2L, so her heart rate will go down from 120 by about 30*0.5 = 15, so her heart rate will be about 105.