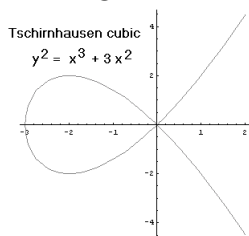


## MIDTERM #2

1. The curve  $y^2 = x^3 + 3x^2$  is called the Tschirnhausen cubic (named after Ehrenfried Walther von Tschirnhaus, who hung out with Leibniz and invented European porcelain). Find an equation of the tangent line to his curve at the point  $(1, -2)$ .



2. Air is being pumped into a spherical balloon. At time  $t$  seconds, the volume of the balloon is  $V(t)$  (in  $\text{cm}^3$ ) and the radius of the balloon is  $r(t)$  (in cm).
- Explain in words the meanings of the derivatives  $dV/dr$  and  $dV/dt$ .
  - If at time  $t = 5$  the radius of the balloon is 10 cm and the radius is increasing at a rate of 3 cm/second, how fast is the volume increasing at time  $t = 5$ ? (REMINDER: The volume of a sphere is  $V = \frac{4}{3}\pi r^3$ .)
3. What is the tangent line approximation to  $f(x) = e^{3x}$  near  $x = 0$ ?
4. Find the absolute maximum and minimum values of  $f(x) = x^3 - 3x^2 + 1$  on the interval  $[-1/2, 4]$ .
5. Farmer Francine wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Fencing costs \$1 per foot. How can she do this so as to minimize the cost of the fence?