Math 118 Solutions to 2nd Midterm

1. What are the maximum and minimum values of the function $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$, and where do they occur?

Check endpoints and critical points. $f'(x) = 3x^2 - 3$, **which exists everywhere, so set** $f'(x) = 0$: $3x^2 - 3 = 0$, **or** $x^2 = 1$, **or** $x = \pm 1$. **Only** $x = 1$ **is in the interval** $[0, 3]$.

x	$f(x)$
0	**1**
1	**-1**
3	**19**

So the maximum is 19, which occurs at $x = 3$, **and the minimum is -1, which occurs at** $x = 1$.

2. Use linear approximation to estimate $\ln(0.9)$. (Hint: $a = 1$ may be a good choice.)

$\mathbf{f(x) \approx f(a) + f'(a)(x - a)}$. **Here,** $a = 1$ **and** $f'(x) = 1/x$, **so** $f(a) = \ln(1) = 0$ **and** $f'(a) = 1/1 = 1$, **and we have** $\ln(x) \approx 0 + 1(x - 1) = x - 1$. **So** $\ln(0.9) \approx 0.9 - 1 = -0.1$

3. Evaluate the following limits, or state that they do not exist.

(a) $\lim_{x\to 0} \frac{x^2}{\sin x}$

(b) $\lim_{x\to 0} \frac{\cos x}{x}$

(a) This is 4.7.5 from the text. Plug in and get 0/0, so we use L'Hopital: $\lim_{x\to 0} \frac{x^2}{\sin x} = \lim_{x\to 0} \frac{2x}{\cos x} = 0/1 = 0$.

(b) this is 4.7.27 from the text. Plug in and get 1/0, so the limit doesn't exist (it diverges to infinity).

4. (a) If r is the radius of a circle with area A and the circle expands as time passes, find dr/dt in terms of dA/dt.

(b) Suppose delicious Vermont maple syrup leaks from its bottle and spreads in a circular pattern. If the area of the syrup spill increases at a constant rate of 2 cm^2/second, how fast is the radius of the spill increasing when the radius is 30 cm?

(a) $\mathbf{A = \pi r^2}$**, so** $\mathbf{dA/dt = 2\pi r\, dr/dt}$**, and** $\mathbf{dr/dt = (dA/dt)/(2\pi r)}$**.**

(b) Plug into $\mathbf{dr/dt = (dA/dt)/(2\pi r)}$**:** $\mathbf{dr/dt = 2/(2\pi 30) = 1/(30\pi)}$ **cm/sec.**

5. Jungle Jane sells robotic monkeys over the internet. To sell q robomonkeys per month, she needs to set the price at $p = 125 - \frac{q}{25}$ dollars per robomonkey. If she has fixed costs of \$10,000 per month, and each robomonkey costs her \$25 to make, how many robomonkeys should she make each month to maximize her profit?

Profit = revenue - cost, or $\pi = R - Q$. **The revenue is** $R = pq = (125 - \frac{q}{25})q = 125q - \frac{q^2}{25}$, **and the cost is** $C = 10,000 + 25q$, **so the profit is** $\pi = 125q - \frac{q^2}{25} - 10,000 - 25q = 100q - \frac{q^2}{25} - 10,000$. **The endpoints are** $q = 0$ **and** $q \to \infty$. **Next, we find the critical points.** $\pi' = 100 - \frac{2q}{25}$, **which exists everywhere, so we set it equal to zero:** $100 - \frac{2q}{25} = 0$, **or** $100 = \frac{2q}{25}$, **or** $2500 = 2q$, **or** $1250 = q$. **This is the only critical point, and** $\pi'' = 2/25 > 0$, **so the maximum must occur there: she should make 1250 robomonkeys each month.**

EXTRA CREDIT Draw a robomonkey.