1. What are the maximum and minimum values of the function \( f(x) = x^3 - 3x + 1 \) on the interval \([0, 3]\), and where do they occur?

   Check endpoints and critical points. \( f'(x) = 3x^2 - 3 \), which exists everywhere, so set \( f'(x) = 0 \): \( 3x^2 - 3 = 0 \), or \( x^2 = 1 \), or \( x = \pm 1 \). Only \( x = 1 \) is in the interval \([0, 3]\).

   \[
   \begin{array}{c|c|c}
   x & f(x) & 0 \quad 1 \quad 3 \\ 
   \hline
   0 & 1 \\ 
   1 & -1 \\ 
   3 & 19 \\ 
   \end{array}
   \]

   So the maximum is 19, which occurs at \( x = 3 \), and the minimum is -1, which occurs at \( x = 1 \).

2. Use linear approximation to estimate \( \ln(0.9) \). (Hint: \( a = 1 \) may be a good choice.)

   \[
   f(x) \approx f(a) + f'(a)(x - a). \quad \text{Here, } a = 1 \quad \text{and} \quad f'(x) = 1/x, \quad \text{so} \quad f(a) = \ln(1) = 0 \quad \text{and} \quad f'(a) = 1/1 = 1, \quad \text{and we have} \quad \ln(x) \approx 0 + 1(x - 1) = x - 1. \quad \text{So} \quad \ln(0.9) \approx 0.9 - 1 = -0.1
   \]

3. Evaluate the following limits, or state that they do not exist.
   
   (a) \( \lim_{x \to 0} \frac{x^2 \sin x}{\cos x} \)
   
   (b) \( \lim_{x \to 0} \frac{2x}{\cos x} \)

   (a) This is 4.7.5 from the text. Plug in and get 0/0, so we use L'Hopital:
   
   \[
   \lim_{x \to 0} \frac{x^2 \sin x}{\cos x} = \lim_{x \to 0} \frac{2x}{\cos x} = 0/1 = 0.
   \]

   (b) this is 4.7.27 from the text. Plug in and get 1/0, so the limit doesn’t exist (it diverges to infinity).

4. (a) If \( r \) is the radius of a circle with area \( A \) and the circle expands as time passes, find \( dr/dt \) in terms of \( dA/dt \).

   (b) Suppose delicious Vermont maple syrup leaks from its bottle and spreads in a circular pattern. If the area of the syrup spill increases at a constant rate of 2 cm\(^2\)/second, how fast is the radius of the spill increasing when the radius is 30 cm?

   (a) \( A = \pi r^2 \), so \( dA/dt = 2\pi r \, dr/dt \), and \( dr/dt = (dA/dt)/(2\pi r) \).

   (b) Plug into \( dr/dt = (dA/dt)/(2\pi r) \): \( dr/dt = 2/(2\pi 30) = 1/(30\pi) \) cm/sec.

5. Jungle Jane sells robotic monkeys over the internet. To sell \( q \) robomonkeys per month, she needs to set the price at \( p = 125 - \frac{q}{25} \) dollars per robomonkey. If she has fixed costs of $10,000 per month, and each robomonkey costs her $25 to make, how many robomonkeys should she make each month to maximize her profit?

   Profit = revenue - cost, or \( \pi = R - Q \). The revenue is \( R = pq = (125 - \frac{q}{25})q = 125q - \frac{q^2}{25} \), and the cost is \( C = 10,000 + 25q \), so the profit is \( \pi = 125q - \frac{q^2}{25} - 10,000 - 25q = 100q - \frac{q^2}{25} - 10,000 \). The endpoints are \( q = 0 \) and \( q \to \infty \). Next, we find the critical points. \( \pi' = 100 - \frac{2q}{25} \), which exists everywhere, so we set it equal to zero: \( 100 - \frac{2q}{25} = 0 \), or \( 100 = \frac{2q}{25} \), or \( 2500 = 2q \), or \( 1250 = q \). This is the only critical point, and \( \pi'' = 2/25 > 0 \), so the maximum must occur there: she should make 1250 robomonkeys each month.

EXTRA CREDIT: Draw a robomonkey.