Math 118 December 2005 Name:________________

FINAL EXAM

This exam is 6 pages long (counting this one); check that you have all the pages. Show your work. Correct answers with no justification may receive little or no credit. No calculators, notes, or books are allowed. No unnecessary simplification is required. Use the backs of pages if you run out of space, make sure that I can find your answers, and THINK JOYFULLY.

PROBLEM	POINTS	SCORE
1	10	
2	15	
3	20	
4	15	
5	20	
6	20	
EXTRA CREDIT	2	
TOTAL	100	

1. (10 pts) My friend Duncan has decided to enter a donut-eating contest. Let $E(x)$ be the number of minutes that it takes him to eat x pounds of donuts. He has discovered, through months of delicious training and experimentation, that $E(3) = 15$ and $E'(3) = 6$. Roughly how long would it take him to eat 2.5 pounds of donuts? (In other words, use linear approximation to estimate $E(2.5)$.)

$$E(x) \approx E(a) + E'(a)(x-a)$$

Here, use $a = 3$ So

$$E(2.5) \approx E(3) + E'(3)(2.5-3)$$

$$= 15 + 6\left(-\tfrac{1}{2}\right)$$

$$= 15 - 3$$

$$= 12 \text{ minutes}$$

(Don't forget problem 1 on the previous page!)

2. (15 pts) Compute the following derivatives:

(a) $\frac{d}{dx}(3^x + \ln 3)$ $\quad (\ln 3)3^x + 0 = (\ln 3)3^x$

(b) $\frac{d}{dx}\left(\int_0^x \cos(2t)\,dt\right)$ Use FT of C: $\cos(2x)$

(c) $\frac{d}{dx}(\ln(x^2+1))$ Chain rule: $\frac{1}{x^2+1}\cdot(x^2+1)' = \frac{2x}{x^2+1}$

(d) $\frac{d}{dt}(3te^{t^2})$ product rule: $3e^{t^2} + 3t(2te^{t^2})$

$= (3+6t^2)e^{t^2}$

(e) $\frac{d}{dt}\left(\frac{2t^5}{1+3t}\right)$ quotient rule $\quad \frac{(1+3t)(10t^4) - 2t^5(3)}{(1+3t)^2}$

$= \frac{10t^4 + 30t^5 - 6t^5}{(1+3t)^2}$

$= \frac{10t^4 + 24t^5}{(1+3t)^2}$

3. (20 pts) My uncle, Donald Uriah Tiberius King, has opened a donut store called Donut Czar. He needs your help to figure out how much he should charge for each donut. He pays \$500 a day in fixed costs (rent, wages, bribing health inspectors, etc.). In addition, it costs him \$0.20 to make each donut. After consulting with my brother the economist, Uncle Don has determined that if he charges $\$p$ for each donut, he will sell $x = 5000 - 5000p$ donuts per day.

(a) Find a formula for the daily cost function $C(x)$, which measures the cost (in dollars) of producing x donuts in a day.

(b) Find a formula for the daily revenue R as a function of x (so your answer $R(x)$ should have only x's in it, and no p's).

(c) How many donuts should he make per day to maximize profit?

(d) How much should he charge per donut?

(e) How much money will he make per day?

$C(x) = 500 + \frac{1}{5}x$

b) $R =$ (price)(quantity sold) $= px$. Since $x = 5000 - 5000p$, $p = 1 - \frac{x}{5000}$,

So $R(x) = \left(1 - \frac{x}{5000}\right)x = x \quad \frac{x^2}{5000}$

profit $\pi(x) = R(x) - C(x) = x - \frac{x^2}{5000} - 500 - \frac{1}{5}x$

$\pi(x) = \frac{4}{5}x - \frac{x^2}{5000} - 500$

Interval: $0 \le x < \infty$

Find the critical pts: $\pi'(x) = \frac{4}{5} - \frac{x}{2500}$

Always exists, so set $= 0$ $\quad \frac{4}{5} - \frac{x}{2500} = 0$, or $x = 2000$

x	$\pi(x)$
0	-500
$\to\infty$	$\to -\infty$ (x^2 term dominates)
2000	$\frac{4}{5}\cdot 2000 - \frac{4{,}000{,}000}{5000} - 500 = 1600 - 800 - 500 = 300$

So make 2000 donuts per day

d) $p = 1 - \frac{x}{5000}$, so when $x = 2000$, $p = 1 - \frac{2000}{5000} = 3/5 = 60¢$

e) $\pi(2000) = \$300$

4. (15 pts) Compute the following integrals.

(a) $\displaystyle\int_0^1 \cos(\pi x)\, dx$ $\quad \frac{1}{\pi} \sin \pi \Big|_0^1 \quad \frac{1}{\pi} \sin \pi \quad \frac{1}{\pi} \sin 0 \quad 0 - 0 \quad 0$

(b) $\displaystyle\int (6x^2 - 6x + 2)\, dx$ $\quad 3 \quad 3x \quad dx$

(c) $\displaystyle\int^1 |x|\, dx$ Easiest to draw picture

Area $\frac{1}{2} \cdot 2 \cdot \quad \frac{1}{2}$

(d) $\displaystyle\int - \, dx$ $\quad \ln |x| + C$

(e) $\displaystyle\int_{\ln 3}^{\ln 6} 8e^t\, dt$ $\quad 8e \Big|_{\ln 3}^{\ln 6} \quad 8e^{\ln 6} \quad 8e^{\ln 3}$

$8 \cdot 6 \quad 8\ 3 \quad 24$

5. (20 pts) It is the year 2035, and you're about to be inaugurated as president of Agnes Scott College. Unfortunately, you've overslept. Skipping breakfast, you hop into your nuclear-powered car and speed toward campus. Suddenly, you see a Donut Czar bakery on the side of the road 550 ~~450~~ feet ahead of you. Hoping to grab a snack before the ceremony, you slam on the brakes. Will the sudden deceleration send you headfirst through the windshield? Will you be able to stop before you pass the Donut Czar? Perhaps math, along with the following data about your car's speed, can help answer these questions. (Your speed decreases throughout the 8 seconds it takes you to stop, although not necessarily at a uniform rate.)

Time t since brakes applied (sec)	0	2	4	6	8
Speed $v(t)$ (ft/sec)	100	80	50	20	0

(a) Estimate the car's acceleration at time $t = 1$ second after the brakes are applied. What are the units?

(b) Use a left-hand Riemann sum to estimate the distance traveled between the time the brakes are applied to the time the car stops (in other words, to estimate $\int_0^8 v(t)\,dt$).

(c) Use a right-hand Riemann sum to estimate the distance traveled between the time the brakes are applied to the time the car stops (in other words, to estimate $\int_0^8 v(t)\,dt$).

(d) Which of the following statements can you justify from the information given? Explain.
   (i) Your car stopped before passing the Donut Czar.
   (ii) The data are inconclusive; your car may or may not have passed the Donut Czar.
   (iii) Your car passed the Donut Czar.

Acc. is the deriv of velocity $a(1) = v'(1) \approx \frac{v(2)-v(0)}{2} = \frac{80-100}{2} = \frac{-20}{2} = -10$

Accelerating at a rate of -10 ft/sec^2

$\int_0^8 v(t)\,dt \approx 100\cdot 2 + 80\cdot 2 + 50\cdot 2 + 20\cdot 2 = 500$ ft

$\int_0^8 v(t)\,dt \approx 80\cdot 2 + 50\cdot 2 + 20\cdot 2 + 0\cdot 2 = 300$ ft

Since $v(t)$ is decreasing, we know that the left-hand Riemann sum is an overestimate, so the stopping distance is $<$ 500 ft, so

i) The car stopped before passing the Donut Czar.

6. (20 pts) A delicious hot Kremey Krisp donut is taken out of the oven and placed on your plate to cool. The temperature H, in degrees Celsius, of the donut t minutes after it was taken out of the oven is given by $H = f(t)$.

(a) Explain in words the meanings of the following equations.

(i) $f(20) = 100$

After 20 minutes, the donut's at 100°.

(ii) $f^{-1}(250) = 1$

The donut reaches 250° after 1 minute.

(b) What are the units of $f'(20)$? What is its practical meaning in terms of the temperature of the delicious donut? Do you expect it to be positive or negative, and why?

Units: $\frac{\text{degrees}}{\text{minute}}$. Roughly, it's how much hotter the donut will be after one more minute. It should be negative, since the temp. is decreasing.

(c) What are the units of $(f^{-1})'(250)$? What is its practical meaning in terms of the temperature of the delicious donut?

Units: $\frac{\text{minutes}}{\text{degree}}$. Roughly, it's how long it will take for the temp. to increase one more degree.

EXTRA CREDIT What's your favorite thing about calculus?