Math 5 December 16, 2002 Name:________________________

Your professor's name:________________________

FINAL EXAM

This exam is 9 pages long; check that you have all the pages. Show your work. Correct answers with no justification may receive little or no credit. No calculators, notes, or books are allowed. No uncalled-for simplification is required. Use the backs of pages if you run out of space, and make sure that we can find your answers.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	15	
4	15	
5	27	
6	12	
7	10	
8	16	
9	20	
10	15	
11	30	
TOTAL	200	

(1) (20 pts) For each of the following functions, compute the derivative with respect to x.

(a) $(x+1)^3 \sin x$

(b) $\dfrac{\ln x^2}{3x}$

(c) $e^{\cos x^2}$

(d) $\displaystyle\int_x^1 \ln(1+e^{t^4})\,dt$

(e) $\displaystyle\int_1^2 e^{\cos x}\,dx$

(2) (20 pts) Compute the following integrals:

(a) $\displaystyle\int_0^1 x^{11}\,dx$

(b) $\displaystyle\int \sin \pi t\,dt$

(c) $\displaystyle\int (e^x + 2x + 2)\,dx$

(d) $\displaystyle\int_{-3}^{-2} \frac{1}{x}\,dx$

(3) (15 pts) Use the definition of derivative to compute $f'(2)$, where $f(t) = 5t^2 + 7$.

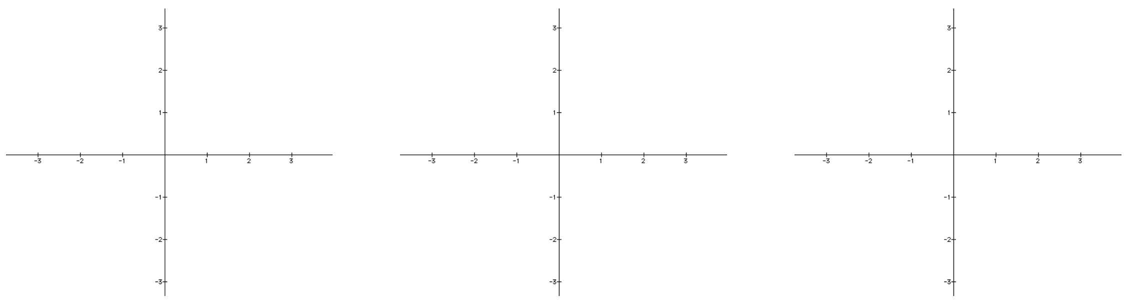
(4) (15 pts) Let $f(T)$ be the number of minutes it takes for an ice cube to melt if the temperature of the air around it is T° Fahrenheit.

(a) (10 pts) Explain, in words, the meaning of the derivative $\dfrac{df}{dT}$.

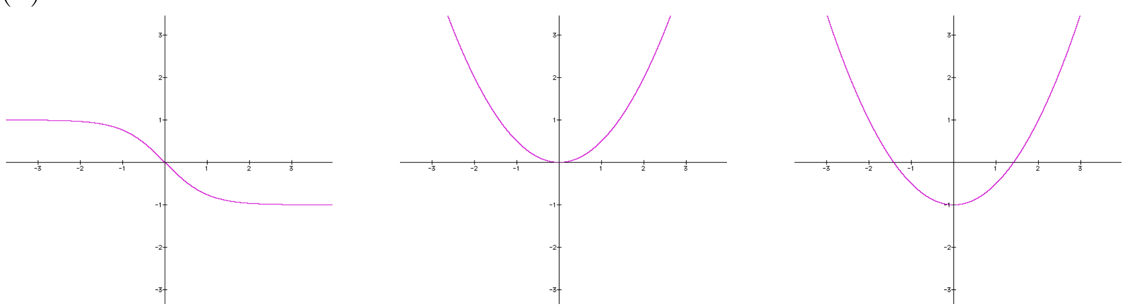
(b) (5 pts) Do you expect $\dfrac{df}{dT}$ to be positive or negative? Why?

(5) (27 pts) Here are the graphs of 3 derivative functions $y = f'(x)$. In each case, make a rough sketch of the graph of $y = f''(x)$ below it and the graph of the particular antiderivative $y = f(x)$ for which $f(0) = 0$ above it.

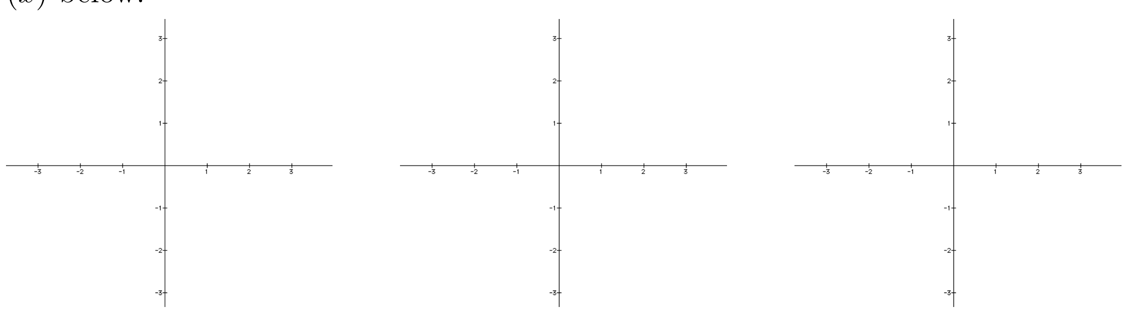
$f(x)$ below:



$f'(x)$ below:



$f''(x)$ below:



(6) (12 pts) I have \$500 in the bank now, and my balance after t days is $500e^{0.01t}$. What is my average balance over the first 30 days (i.e., from $t = 0$ to $t = 30$)?

(7) (10 pts) Consider the hyperbola $x^2 - y^2 = 7$.

(a) What is the slope of the tangent line to the hyperbola at the point $(4, 3)$?

(b) Find an equation for the tangent line at that point.

(8) (16 pts) Compute the following limits, or explain why they do not exist. Quote any results that you use.

(a) $\displaystyle \lim_{x \to 0} \frac{\sin x}{x^2 + 2x}$

(b) $\displaystyle \lim_{x \to \infty} \frac{\sin x}{x^2 + 2x}$

(9) (20 pts) A flying saucer traveling 500 miles per hour at an altitude of 3 miles flew directly over Uncle Ant. Let x and y be as labeled in the figure.

(a) (4 pts) Find an equation relating x and y.

(b) (4 pts) Find the value of x when y is 5 miles.

(c) (12 pts) How fast is the distance from Uncle Ant to the flying saucer changing at the time when the saucer is 5 miles from Uncle Ant? That is, what is $\dfrac{dy}{dt}$ at the time when $y = 5$?

(10) (15 pts)

(a) (10 pts) Use linear approximation or one iterate of Newton's method to estimate $\sqrt{10}$.

(b) (5 pts) Use the picture to explain why Newton's method will fail to find a solution of $f(x) = 0$ if you start with the initial guess $x = x_0$.

(11) (30 pts) A canvas wind shelter for the beach is to have a back, two *square* sides, and a top. You have 96 square feet of canvas available to build the shelter.

(a) (10 pts) Let x be the height of the shelter and y the length.

(i) What is the surface area of the shelter in terms of x and y?

(ii) What is the volume of the shelter in terms of x and y?

(b) (20 pts) Find the dimensions of the shelter with maximum space inside (i.e., with maximum volume).