Math 119 Midterm #2 Solutions

1. Consider the region bounded by \( y = x^2 \), the \( y \)-axis, and the line \( y = 4 \), with \( x \geq 0 \). Find the volume of the solid obtained by rotating the region about the \( y \)-axis. (Careful: Be sure to rotate about the \( y \)-axis, not the \( x \)-axis.)

The cross sections are disks, so the volume is \( \int_0^4 \pi x^2 \, dy \). We solve for \( x \) in terms of \( y \) to get \( x = \sqrt{y} \), so we plug in and get \( V = \int_0^4 \pi y \, dy = 8\pi \).

2. Use a second-degree Taylor polynomial to approximate \( \ln(1.1) \).

The Taylor polynomial for \( \ln(x) \) at \( x = 1 \) is \( P_2(x) = (x - 1) - \frac{1}{2}(x - 1)^2 \), so \( \ln(1.1) \approx P_2(1.1) = .1 - \frac{1}{2}(.1)^2 = 0.095 \).

3. For several years I’ve been recording the amount of delicious chocolate milk that I drink each day. The data show that the density function of these amounts is given approximately by the function \( p(x) \) (where \( x \) is the amount in gallons). Set up integrals to answer the following questions.
   (a) On what proportion of days do I drink between 4 and 5 gallons of delicious chocolate milk?
   (b) On what proportion of days do I drink 8 or more gallons of delicious chocolate milk?
   (c) On average, how many gallons do I drink per day?

   a) \( \int_4^5 p(x) \, dx \)  
   b) \( \int_8^\infty p(x) \, dx \)  
   c) \( \int_0^\infty xp(x) \, dx \)

4. This problem deals with the questions of estimating the cumulative effect of a tax cut on a country’s economy. Suppose the government proposes a tax cut totaling $100 million. We assume that all the people who have extra money to spend would spend 80% of it and save 20%. Thus, of the extra income generated by the tax cut, $100(0.8) million = $80 million would be spent and so become extra income to someone else. Assume that these people also spend 80% of their additional income, or $80(0.8) million, and so on. Calculate the total additional spending created by such a tax cut.

   (This is problem 9.2.31 from your homework.)

   This is a geometric series: \((.8)100 + (.8)^2100 + \cdots = 80 \frac{1}{1 - .8} = 400\) million dollars.

5. Find the radius of convergence of the power series \( \sum_{n=1}^\infty \frac{x^n}{n \cdot 3^n} \).

   The radius of convergence is \( \lim_{n \to \infty} \frac{a_n}{a_{n+1}} = \lim_{n \to \infty} \frac{\frac{1}{n \cdot 3^n}}{\frac{1}{(n+1) \cdot 3^{n+1}}} = \lim_{n \to \infty} \frac{3 \cdot (n+1)}{n} = 3 \).