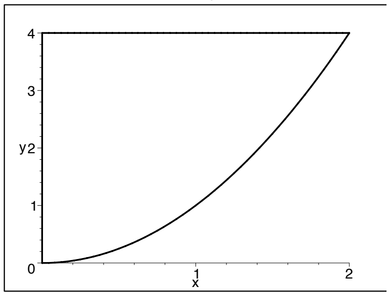
Math 119 Midterm #2 Solutions

1. Consider the region bounded by $y = x^2$, the y-axis, and the line $y = 4$, with $x \geq 0$. Find the volume of the solid obtained by rotating the region about the y-axis. (**Careful:** Be sure to rotate about the y-axis, not the x-axis.)



The cross sections are disks, so the volume is $\int_0^4 \pi x^2\, dy$**. We solve for** x **in terms of** y **to get** $x = \sqrt{y}$**, so we plug in and get** $V = \int_0^4 \pi y\, dy = 8\pi$**.**

2. Use a second-degree Taylor polynomial to approximate $\ln(1.1)$.

The Taylor polynomial for $\ln(x)$ **at** $x = 1$ **is** $P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$**, so** $\ln(1.1) \approx P_2(1.1) = .1 - \frac{1}{2}(.1)^2 = 0.095$**.**

3. For several years I've been recording the amount of delicious chocolate milk that I drink each day. The data show that the density function of these amounts is given approximately by the function $p(x)$ (where x is the amount in gallons). Set up integrals to answer the following questions.

(a) On what proportion of days do I drink between 4 and 5 gallons of delicious chocolate milk?
(b) On what proportion of days do I drink 8 or more gallons of delicious chocolate milk?
(c) On average, how many gallons do I drink per day?

a) $\int_4^5 p(x)\, dx$ **b)** $\int_8^\infty p(x)\, dx$ **c)** $\int_0^\infty xp(x)\, dx$

4. This problem deals with the questions of estimating the cumulative effect of a tax cut on a country's economy. Suppose the government proposes a tax cut totaling \$100 million. We assume that all the people who have extra money to spend would spend 80% of it and save 20%. Thus, of the extra income generated by the tax cut, \$100(0.8) million = \$80 million would be spent and so become extra income to someone else. Assume that these people also spend 80% of their additional income, or \$80(0.8) million, and so on. Calculate the total additional spending created by such a tax cut.

(This is problem 9.2.31 from your homework.)

This is a geometric series: $(.8)100 + (.8)^2 100 + \cdots = 80\frac{1}{1-.8} = 400$ **million dollars.**

5. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 3^n}$.

The radius of convergence is $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{\frac{1}{n \cdot 3^n}}{\frac{1}{(n+1)\cdot 3^{n+1}}} = \lim_{n\to\infty} \frac{3 \cdot (n+1)}{n} = 3$**.**