Math 119 Practice Final Solutions

1. Chapter 6 Review (p. 306), #63.
2. Chapter 7 Review (p. 361), #1-146 as needed.
3. Chapter 9 Review (p. 471), #23-41 odd.
4. Chapter 11 Review (p. 595), #3, 5, 7, 25, 27.
5. Use a second-degree Taylor polynomial to approximate $\ln 1.3$. How big might the difference between your approximation and the real answer be?

Let's approximate near $a = 1$. The second-degree Taylor polynomial there is $P_2(x) = 0 + (x-1) - (x-1)^2/2$, so $\ln(1.3) \approx P_2(1.3) = 0.3 - (0.3)^2/2 = 0.255$. We know that $\ln'''(x) = \frac{2}{x^3}$. The biggest this gets on the interval [1,1.3] is at 1, when it's 2. So the error is $\leq \frac{2}{3!}(0.3)^3 = .009$.

6. Suppose that the government spends an extra \$1 billion. Assume that each business and individual saves 25% of its income and spends the rest, so of the initial \$1 billion, 75% is respent by individuals and businesses. Of that amount, 75% is spent, and so on. What is the total increase in spending due to the government action? (This is called the *multiplier effect* in economics.)

Total increase is $10^9 + (.75)10^9 + (.75)(.75)10^9 + \cdots = 10^9(1 + .75 + .75^2 + \dots) = 10^9(\frac{1}{1-.75}) = 4 \cdot 10^9$.

7. Find the length of the curve $y = x^{3/2}$ from $x = 1$ to $x = 4$.

Arc length is

$$\int_1^4 \sqrt{1 + (dy/dx)^2}\, dx = \int_1^4 \sqrt{1 + (\frac{3}{2}x^{1/2})^2}\, dx = \int_1^4 \sqrt{1 + \frac{9}{4}x}\, dx = \frac{8}{27}(1 + \frac{9}{4}x)^{3/2}|_1^4 \approx 7.63.$$

8. Let R be the region bounded by the curve $y = x - x^2$ and the x-axis.
(a) Find the area of R.
(b) Find the volume of the solid generated by revolving the region R about the x-axis.

(a) The curve intersects the x-axis at 0 and 1, so the area is $\int_0^1 (x - x^2)\, dx = 1/6$.

(b) The volume is $\int_0^1 \pi(x - x^2)^2\, dx + \int_0^1 \pi(x^4 - 2x^3 + x^2)\, dx = \pi/30$.

9. Let $z = 2 - 2i$.
(a) Write z in the form $re^{i\theta}$.
(b) Find $\bar{z}$.
(c) Find z^2.
(d) Find $|z|$.
(e) Find $1/z$.

(a) $2\sqrt{2}e^{-\pi i/4}$ (b) $2 + 2i$ (c) $-8i = 8e^{-\pi i/2}$ (d) $2\sqrt{2}$ (e) $(2 + 2i)/(2\sqrt{2})^2 = 1/4 + 1/4i = \frac{1}{2\sqrt{2}}e^{\pi i/4}$.

10. Evaluate (a) $\int_1^3 \frac{dx}{(x-1)^{1/3}}$ and (b) $\int_9^\infty \frac{x}{\sqrt{1+x^2}}\, dx$, or show that they do not exist.

(a) $\lim_{b\to 1^+} \int_b^3 \frac{dx}{(x-1)^{1/3}} = \lim_{b\to 1^+} \frac{3}{2}(x-1)^{\frac{2}{3}}|_b^3 = \frac{3}{2}2^{2/3} - 0 = 3/\sqrt[3]{2}$ (b) $\lim_{b\to\infty} \int_9^b \frac{x}{\sqrt{1+x^2}}\, dx = \lim_{b\to\infty} (1 + x^2)^{\frac{1}{2}}|_9^b \to \infty$, so DNE.

11. The lifetime of a cellphone, measured in months, can be modeled by a random variable X with an *exponential distribution*, which has probability density function $f(t) = 0.01e^{-0.01t}$ $(t \geq 0)$. What is the probability that a randomly selected cellphone lasts for at least 20 months?

This is the probability that the lifetime is 20 or greater: $P(X \geq 20) = \int_{20}^{\infty} 0.01e^{-0.01t}\,dt = e^{-0.2} \approx 0.819$**.**

12. A piece of wire, stretching from $x = 0$ to $x = 10$, has density $\delta(x) = 3x^2$. Find (a) the mass of the wire, and (b) the center of mass of the wire.

(a) $\int_0^{10} \delta(x)\,dx = 1000$**. (b)** $\frac{\int_0^{10} x\delta(x)\,dx}{\textbf{total mass}} = \frac{7500}{1000} = 7.5$ **cm.**

13. If q Dancin' Scottie dolls are to be sold, the price must be set at $p = D(q) = 3e^{\frac{-q}{1{,}000{,}000}}$ dollars.

(a) If 18,000 are sold, what's the price?

(b) If the price is \$1, how many are sold?

(c) What's the consumer surplus, if the equilibrium price is \$1?

(a) $D(18{,}000) = 2.95$ **dollars (b) Plug in** $p = 1$ **and solve for** q**: 1,098,612 (c) When** $p^* = 1$**,** $q^* = 1{,}098{,}612$ **(from part b), so the consumer surplus is** $\left(\int_0^{q^*} D(q)\,dq\right) - p^*q^* = \left(\int_0^{1098612} 3e^{\frac{-q}{1{,}000{,}000}}\,dq\right) - 1 \cdot 1098612 = -3000000(e^{-\frac{1098612}{1000000}} - 1) - 1098612 = 901{,}387.71$

14. Calculate $\frac{d}{dx}\int_5^{e^x} \cos(t^3)\,dt$.

Fundamental Theorem of Calculus and chain rule! $\cos((e^x)^3) \cdot \frac{d}{dx}(e^x) = e^x \cos(e^{3x})$

15. We want to study the populations of whales in the ocean and of mold on the bread in my refrigerator. We know that whales live in the vast, vast ocean and have plenty of room and lots of delicious plankton to eat. At the same time, though, they're so spread out that if there aren't very many of them, they won't be able to find each other in order to breed.

We also know that mold reproduces asexually (i.e., a mold cell doesn't need another mold cell to reproduce). On the other hand, once the mold covers the bread, there'll be no place else for it to live.

(a) Differential equations modeling the two populations $P(t)$ and $Q(t)$ are given below. Which is which? That is, is $P(t)$ the whale population at time t and $Q(t)$ the mold population, or is it the other way around? Explain your reasoning. (k_1 and k_2 are positive constants.)

$$\frac{dP}{dt} = k_1P(P - 10{,}000) \qquad\qquad \frac{dQ}{dt} = k_2Q(10{,}000 - Q)$$

(b) What are the possible long-term behaviors of solutions to the first equation ($\frac{dP}{dt} = k_1P(P - 10{,}000)$)?

(a) Q **decreases if** Q **is large and increases if** Q **is small, so that must be the mold population. Also,** P **increases if** P **is large and decreases if** P **is small, so that must be the whale population.**

(b) (It may help to sketch the slope field.) Equilibria are $P = 0$ **and** $P = 10000$**. If the initial population is below 10000, they die off. If the initial population is above 10000, the population increases without bound.**