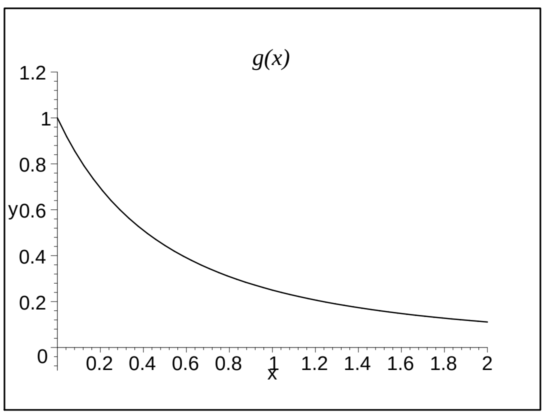
Math 119 Practice First Midterm Solutions

The actual midterm will of course be shorter than this. You may use a calculator, but not any of its calculus functions. Don't expect too many "compute the integral" problems; that material is covered by the online Integral Proficiency Test.

1. Chapter 6 Review (p. 306): #51, 55, 69. Chapter 7 Review (p. 361): #1-132 as needed, 161. Section 8.1 (p. 374): #26.

Answers to odd questions are in the back of the book. 8.1.26: a) $\int_0^4 3\,dx = 12$ **b)** $\int_0^3 (8 - \frac{8}{3}h)\,dx = 12$**.**

2. The left, right, trapezoid, and midpoint rules were used to estimate $\displaystyle\int_0^2 g(x)\,dx$, where g is the function whose graph is shown. The estimates were 0.7811, 0.8632, 0.8675, and 0.9540, and the same number of subintervals were used in each case.



(a) Which rule produced which estimate? Explain.

(b) Between which two estimates does the true value of $\displaystyle\int_0^2 g(x)\,dx$ lie?

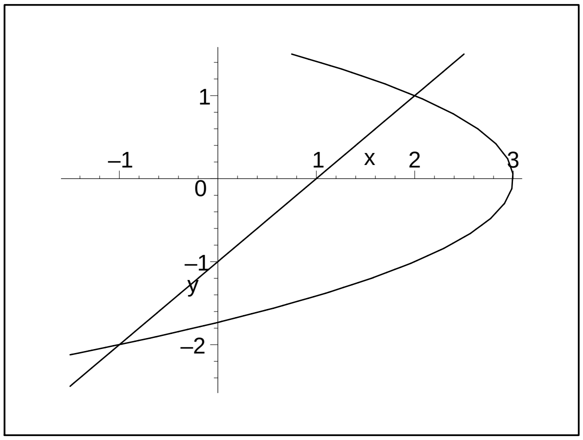
(a) By looking at the slope and concavity of the graph, we see that the left rule overestimates by a lot, the right rule underestimates by a lot, the trapezoid rule overestimates by a little, and the midpoint rule underestimates by a little. So 0.7811 is the right rule, 0.8632 is the midpoint rule, 0.8675 is the trapezoid rule, and 0.9540 is the left rule.

(b) Reasoning as in (a), we see that the real answer is between the midpoint and trapezoid estimates, 0.8632 and 0.8675.

3. Compute $\displaystyle\frac{d}{dx}\left(\int_x^1 \ln t\,dt\right)$.

(This is problem 6.4.19 from your homework.) $\displaystyle\frac{d}{dx}\left(\int_x^1 \ln t\,dt\right) = \frac{d}{dx}\left(-\int_1^x \ln t\,dt\right) = -\ln x$**, by the Fundamental Theorem of Calculus.**

4. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$. (The graphs intersect at the points $(-1, -2)$ and $(2, 1)$, as shown in the figure.)



Use horizontal slices. The width of a slice is $(3 - y^2) - (y + 1) = 2 - y^2 - y$**, so the total area is** $\int_{-2}^{1} (2 - y^2 - y)\, dy = 9/2$**.**

5. For what type of integrand do the Left Sum and Right Sum approximations give the exact value for the integral on any interval? For what type of integrand do the Trapezoid and Midpoint approximations give the exact value for the integral on any interval?

The Left and Right Sums give the exact answer if the integrand is a constant; T & M give the exact answer if it's a straight line.

6. Assuming that the 440 feet is accurate and you neglect air resistance, determine the accuracy of the following paragraph. (A mile is 5280 feet.)

My Journey Beneath the Earth

Condensed from "A Wolverine is Eating My Leg," by Tim Cahill

> I am in Ellison's Cave, about to rappel down Incredible Pit, the second-deepest cave pit in the continental United States. The drop is 440 feet, about what you'd experience from the top of a 40-story building. If you took the shaft in a free fall, you'd accelerate to more than 100 miles an hour and then – about five seconds into the experience – you'd decelerate to zero. And die.

Yup, it's accurate.