Math 311 Practice Final

You *don't* need to memorize the Sharkovskii ordering; I'll give it to you if you need it.

1. Chapter 7 of Devaney (p. 80), #9-15, 21

2. Chapter 9 of Devaney (p. 111), #18

3. Chapter 10 of Devaney (p. 130), #25

4. Chapter 11 of Devaney (p. 151), #8, 16

5. Let $f(x) = ax - x^3$, where the parameter a is a real number.

(a) Find the fixed points and determine for what values of a each fixed point exists.

(b) Investigate the stability of each fixed point.

(c) Discuss the bifurcation that occurs at $a = 1$.

(d) Verify that $\{\sqrt{1+a}, -\sqrt{1+a}$ is a 2-cycle $(a \geq -1)$.

(e) Investigate the stability of this 2-cycle.

6. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (2ax(1 - bx), x^3 + \frac{1}{2}y)$, where the parameters a and b are real numbers.

(a) Let $a > 0$ and $b = 1$. Find the fixed points of F.

(b) Investigate the stability of each fixed point.

(c) For $a = 1$, $b = 0$, show that $(0, 0)$ is a saddle fixed point for F.

(d) For $a = 1$, $b = 0$, calculate the inverse map F^{-1} and show that the sets $U = \{(t, 2t^3/15)\}$ and $S = \{(0, t)\}$ are invariant under F and F^{-1}. Show that U and S are the stable and unstable manifolds, respectively, for F. Are there any homoclinic points for $(0, 0)$?

7. Sketch the proof that the quadratic map $Q_c(x) = x^2 + c$ is chaotic for $c < -(5 + 2\sqrt{5})/4$.

8. (Short answer.) Let X be the space of all possible states of the atmosphere, and let $F : X \to X$ be the map that calculates tomorrow's atmospheric state, given today's atmospheric state as input. What does it mean for weather prediction if F depends sensitively on initial conditions?

9. Define what it means for two dynamical systems to be conjugate, and show that the doubling map on the circle and the tripling map on the circle are not conjugate. (Hint: How many fixed points does each map have?)

10. Show that the map pictured in Figure 5.22(a) on p. 226 of ASY has a point of (prime) period 3.

11. Give examples of a dynamical system that

(a) has a dense orbit but is not chaotic.

(b) has dense periodic points but is not chaotic.

(c) has SDIC but is not chaotic.