Math 321 Algebra Practice Midterm #1

1. Let $H = \{\sigma \in S_n \mid \sigma(1) = 1\}$. Prove that H is not a normal subgroup of S_n for $n \geq 3$.

2. COMPUTATIONAL PROBLEMS

(a) Draw the subgroup diagram for $\mathbb{Z}_{45}$.

(b) List all abelian groups of order 45, up to isomorphism.

(c) Let $G = S_4$ and let $H = \{e, (123), (132)\}$. Compute the left coset gH, where $g = (12)(34)$. (You do not need to show that H is, in fact, a subgroup of G.)

(d) Compute $(G : H)$, the index of H in G, where H and G are given in part (c).

(e) Classify the factor group $(\mathbb{Z}_3 \times \mathbb{Z}_6)/\langle(1, 2)\rangle$ according to the fundamental theorem of finitely generated abelian groups.

(f) Let G be the group of all polynomials with real coefficients. Let $\phi_0 : G \to \mathbb{R}$ be the evaluation homomorphism given by $\phi_0(P(x)) = P(0)$. Find the kernel of ϕ. What is the factor group $G/\ker\phi_0$?

(g) Find the primitive sixth roots of unity.

3. Let G be a group, and let $\phi : G \to G$ be defined by $\phi(g) = g^{-1}$. Is ϕ an isomorphism? If so, prove it; if not, show exactly which properties of an isomorphism fail. (That is, prove which properties do hold and explain which ones don't and why.)

4. Let G be a finite group, and suppose that H and K are subgroups of G.

(a) Prove that $H \cap K$ is a subgroup of G.

(b) Prove that if the order of H is relatively prime to the order of K then $H \cap K = \{e\}$.