1. Recall that the Jefferson apportionment method works as follows. First give each state one representative. Then, give the next representative to the state with the smallest value of $\frac{a_i+1}{p_i}$, where a_i is the number of seats that state i has gotten so far, and p_i is the population of state i. Keep going until all the representatives are apportioned.

(a) Why does this method favor the populous states more than the Adams method, which gives the next representative to the state with the smallest value of $\frac{a_i}{p_i}$?

(b) What is the Alabama Paradox, and why can't it occur with the Jefferson method?

(c) What problem does occur with the Jefferson method (other than favoring the large states)?

(a) Because we're adding $\frac{1}{p_i}$, which is small if the population p_i is big and big if the population is small, to $\frac{a_i}{p_i}$. Thus a large state is more likely to have the smallest value of $\frac{a_i+1}{p_i}$, and get the next seat with the Jefferson method, than it is to have the smallest value of $\frac{a_i}{p_i}$, and get the next seat with the Adams method.

(b) The AP occurs when a state has a larger apportionment at house size h than at house size $h+1$. It can't happen with the Jefferson method because that hands out the first n seats the same way regardless of the total house size. Thus the apportionment at house size $h+1$ is the same as at h, except that one state gets an extra seat.

(c) The Jefferson method can violate quota. For example, if Georgia is entitled to 11.38 seats, it might end up getting 10 or 13.

2. In an experiment, rats are sent through a simple maze consisting of three rooms, A, B, and C. Room A contains a trap, and room B contains delicious cheese. After several experiments it is determined that of the mice who are in room B at a particular time, 90% will still be in Room B nibbling on cheese one minute later, 5% will go to Room A, and 5% to Room C. Of the mice who are in room C at a particular time, 40% will still be in Room C one minute later, 30% will go to Room A, and 30% to Room B. A rat that enters Room A will stay there (since it's trapped like a rat).

(a) Write down a transition matrix describing this situation, and label the absorbing states.

(b) Write down, but *do not solve*, an equation or equations that will tell you how many times on average a rat that starts in Room B will visit Room C before it's trapped.

(a) A is 1, B is 2, C is 3. A is the only absorbing state. The matrix is $\begin{bmatrix} 1 & 0 & 0 \\ .05 & .9 & .05 \\ .3 & .3 & .4 \end{bmatrix}$.

(b) We want the entry in the first row, second column, of the matrix $\mathbf{T} = \left(\mathbf{I} - \begin{bmatrix} .9 & .05 \\ .3 & .4 \end{bmatrix}\right)^{-1}$.

3. Let G be a connected, undirected graph. Recall that an Euler path is a path that includes each edge exactly once, and an Euler tour is an Euler path that starts and ends at the same vertex.

Show that G has an Euler path if and only if it's possible to add one edge (possibly beginning and ending at the same vertex) to G in such a way that the resulting graph has an Euler tour.

$\Rightarrow$ Say our Euler path begins at vertex i and ends at vertex j (where i and j may be the same). Then if we add a new edge from j to i to our graph and Euler path, we get an Euler tour for the new graph.

$\Leftarrow$ If we remove the new edge from the Euler tour of the new graph, we're left with an Euler path of the original graph.