Math 61 **MIDTERM** **October 6, 2004**

Write your answers in your blue book. Show your work. Correct answers with no justification may receive little or no credit. No calculators, notes, or books are allowed. No unnecessary simplification is required. Make sure that I can find your answers.

1. Explain what the Alabama paradox is, and why it can't occur if we use the Jefferson method of apportionment.

2. For reasons that I've never fully understood, when I was growing up my father would often say to me, "Fun's fun, but who wants to die laughing?" Since fun *is* fun, let's calculate the probability that I'll die laughing. Let's say that every minute, I'm either laughing or not. If I'm laughing now, there's a 4/5 probability that I'll be laughing the next minute, a 1/10 probability that I'll be alive but not laughing the next minute, and a 1/10 probability that I'll be dead the next minute (so I died laughing). If I'm alive but not laughing now, there's a 3/10 probability that I'll be laughing the next minute, a 3/5 probability that I'll be alive but not laughing the next minute, and a 1/10 probability that I'll be dead the next minute.

(a) What is the probability that I'll die laughing, given that I'm not laughing now?

(b) What is my expected lifetime, given that I'm not laughing now? What fraction of my remaining minutes can I expect to spend laughing?

3. The registrar is scheduling final exams, and he needs your help. To begin, he has constructed an undirected graph G, defined as follows:

- The vertices are the classes being offered this semester. (So Math 61 is one vertex, and Bio 1 is another.)
- There's an edge between two vertices (classes) if there's at least one student who is enrolled in both classes.

Let's say that there are ten possible exam times (morning, afternoon, and evening on days one, two, and three, and morning on day four). Assign each exam time a different color. Then feasible exam schedules (that is, schedules under which no student has two finals scheduled at the same time) correspond to colorings of the graph (that is, a choice of colors for the vertices such that no two adjacent vertices are the same color).

(a) The registrar would prefer a schedule under which no student has three exams in a 24-hour period. *Given only the graph G* (that is, *not* the list of enrollments that generated it), can you tell whether a given schedule/coloring satisfies this condition? If so, how? If not, are there any circumstances under which you could answer either yes or no for sure?

(b) Okay, now forget about part (a). The registrar also wants to let as many students as possible leave early, that is, to minimize the number of students taking a final on the last day (during exam time 10). Formulate, *but do not solve*, an integer programming problem whose solution will tell him when to schedule each class's exam.

Here are some useful facts and notation:

(i) There are ten exam times, $k = 1, \ldots, 10$.
(ii) There are N different classes, $i = 1, \ldots, N$.
(iii) Each class must be scheduled for exactly one exam time.
(iv) No two classes whose vertices in G are adjacent can be scheduled for the same exam time.
(v) There are e_i students enrolled in class i.
(vi) Classes $1, \ldots, M$ have enrollments of 40 or more. The other classes have enrollments under 40.
(vii) No two classes can have their exams in the same classroom at the same time.
(viii) There are C classrooms available.
(ix) Any classroom can hold any class of under 40 students, but only B of these classrooms can hold a class of 40 or more students.

HINTS: What function are you trying to optimize? What are your constraints? Some useful notation:

$$x_{ik} = \begin{cases} 1, \text{ if class } i \text{ is scheduled for exam time } k, \\ 0 \text{ otherwise.} \end{cases}$$

and

$$a_{ij} = \begin{cases} 1, \text{ if class } i \text{ and class } j \text{ are adjacent in } G, \\ 0 \text{ otherwise.} \end{cases}$$