1. Explain whether the following spaces are vector spaces. (Addition and scalar multiplication for functions are defined by

\[(f + g)(x) = f(x) + g(x), \quad (cf)(x) = c(f(x)).\]

(a) \(V\) is all continuous functions \(f\) on \([0,1]\) such that \(f(0) = 0\).

(b) \(V\) is all continuous functions \(f\) on \([0,1]\) such that \(f(0) = 0\) and \(f(1) = 1\).

(c) \(V\) is all continuous functions \(f\) on \([0,1]\) such that \(f(x) > 0\) for all \(x\).

(d) \(V\) is all continuous functions \(f\) on \([0,1]\) such that \(f(0) = 2f(1)\).

2. Show that, in any vector space,

(a) if \(\vec{u} + \vec{v} = \vec{u} + \vec{w}\), then \(\vec{v} = \vec{w}\).

(b) \(2\vec{v} = \vec{v} + \vec{v}\).

3. We say that the vectors \(\vec{v}_1, \ldots, \vec{v}_m\) are \textit{linearly independent} if the equation

\[c_1\vec{v}_1 + \ldots + c_m\vec{v}_m = \vec{0}\]

has only the solution \(c_1 = \ldots = c_m = 0\); otherwise, we say that they are \textit{linearly dependent}.

(a) When exactly are the vectors \(\begin{bmatrix} a \\ b \end{bmatrix}\) and \(\begin{bmatrix} c \\ d \end{bmatrix}\) linearly independent?

(b) Let \(\vec{v}_1, \ldots, \vec{v}_m\) be linearly dependent. Let \(\vec{u}\) be any other vector. Show that the vectors \(\vec{v}_1, \ldots, \vec{v}_m, \vec{u}\) are also linearly dependent.