Math 16H Worksheet 9/1/2004
Due in class Friday, 9/3.

1. Explain whether the following spaces are vector spaces. (Addition and scalar multiplication for functions are defined by

$$(f+g)(x) = f(x) + g(x),\ (cf)(x) = c(f(x)).)$$

(a) V is all continuous functions f on [0,1] such that $f(0) = 0$.
(b) V is all continuous functions f on [0,1] such that $f(0) = 0$ and $f(1) = 1$.
(c) V is all continuous functions f on [0,1] such that $f(x) > 0$ for all x.
(d) V is all continuous functions f on [0,1] such that $f(0) = 2f(1)$.

2. Show that, in any vector space,
(a) if $\vec{u} + \vec{v} = \vec{u} + \vec{w}$, then $\vec{v} = \vec{w}$.
(b) $2\vec{v} = \vec{v} + \vec{v}$.

3. We say that the vectors $\vec{v}_1, \ldots, \vec{v}_m$ are *linearly independent* if the equation

$$c_1\vec{v}_1 + \ldots c_m\vec{v}_m = \vec{0}$$

has only the solution $c_1 = \cdots = c_m = 0$; otherwise, we say that they are *linearly dependent*.
(a) When exactly are the vectors $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} c \\ d \end{bmatrix}$ linearly independent?
(b) Let $\vec{v}_1, \ldots, \vec{v}_m$ be linearly dependent. Let $\vec{u}$ be any other vector. Show that the vectors $\vec{v}_1, \ldots, \vec{v}_m, \vec{u}$ are also linearly dependent.