

1. (10 points) Find an equation for the tangent line to the curve  $y = 3(x + 1)^2$  at the point  $(-2, 3)$ .

2. (10 points) Newton's law of gravitation states that the gravitational force  $F$  between two bodies of mass  $m_1$  and  $m_2$ , respectively, is given by the equation  $F(r) = G \frac{m_1 m_2}{r^2}$ , where  $r$  is the distance between the centers of mass of the bodies and  $G$  is the gravitational constant ( $G$  is positive). Compute  $\frac{dF}{dr}$ , and explain the physical significance of the sign of the derivative.

3. (12 points) Let  $f(x) = e^{2x}$ .

a) Is  $f(x)$  increasing or decreasing at  $x=1$ ? How do you know?

a) Is  $f'(x)$  increasing or decreasing at  $x=1$ ? How do you know?

a) Let  $n$  be a positive integer (e.g., 1, 2, or 3). What is  $\frac{d^n f}{dx^n}$ ?

4. (12 points) USE THE DEFINITION OF DERIVATIVE to compute the derivative of the function  $f(x) = x^2$ .

5. (12 points) Compute the derivatives of the following functions. YOU DON'T NEED TO SIMPLIFY YOUR ANSWERS.

a)  $f(x) = \frac{\sin x}{x+1}$

b)  $f(x) = (3x^{21} - x^{-7.2}) \cdot 4^x$

c)  $f(x) = \ln(\cos(x^2))$

d)  $f(x) = x^x$

**6.** (12 points) Each of the graphs above shows the position of a particle moving along the x-axis as a function of time,  $0 \leq t \leq 5$ . During this time interval, which particle has:

- a) Constant velocity?
- b) The greatest initial velocity?
- c) The greatest average velocity?
- d) Zero average velocity?
- e) Zero acceleration?
- f) Positive acceleration throughout?

7. (10 points) For each of the following statements, explain whether it's true or false. If it's false, give an example showing that it's false.

a) If  $f'(x_0) = 0$ , then  $f''(x_0) = 0$  as well.

b) If  $f$  and  $g$  are differentiable at  $x_0$ , then  $\frac{f}{g}$  is differentiable at  $x_0$  as well.

c) If  $f$  is differentiable, then it's continuous.

d) If  $f'(x) = g'(x)$  for every  $x$ , then  $f(x) = g(x)$  for every  $x$ .

e) If  $f(x) = g(x)$  for every  $x$ , then  $f'(x) = g'(x)$  for every  $x$ .

**8.** (12 points) Uncle Ant is flying a kite. The kite is at a constant altitude of 400 feet, being blown away from Uncle Ant horizontally at a constant speed. If it's directly over his head at 9:00 am, and it's moved 800 feet horizontally by 1:00 pm, how fast is the string being payed out at the time when 500 feet of string is already out? (Assume that the string is perfectly taut, so that it forms a straight line, and that Uncle Ant's height is 0.)

**9.** (10 points) DO ONLY ONE of the following problems. I will grade only one of them, so if you try both, be sure to tell me which one you want graded.

- a) Prove the quotient rule. (You may use any other rules that we've learned.)
- b) State the definition of the derivative of the function  $f(x)$  at the point  $x_0$ . Using a picture, explain (in words) why the slope of the tangent line to the graph  $y=f(x)$  at the point  $x_0$  is given by that definition. Be sure to label your picture well.