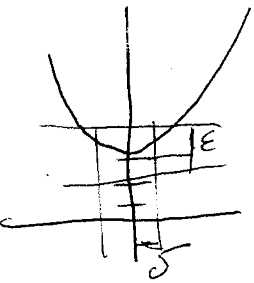
Math 5 Calculus Wiseman 9/4/02

Limits Worksheet

(1) Let $f(x) = x^2 + 3$. We think that $\lim_{x\to 0} f(x) = 3$, and we want to prove it from the definition. Here's how: If I give you any positive number ϵ, you have to be able to find a positive number δ which is small enough that if x is within δ of 0, then $f(x) = x^2 + 3$ is within ϵ of 3. Find such a δ. (Your answer will involve ϵ.)



Need: $3-\varepsilon < x^2+3 < 3+\varepsilon$

$-\varepsilon < x^2 < \varepsilon$

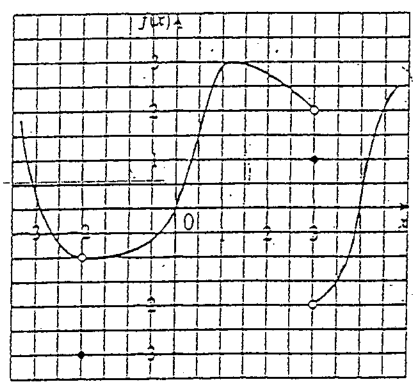
x^2 is never negative, so $-\varepsilon < x^2$ automatically.

So I need to solve $x^2 < \varepsilon$, or $-\sqrt{\varepsilon} < x < \sqrt{\varepsilon}$.

In other words, I need x to be within

$\delta = \sqrt{\varepsilon}$ of 0.

(2) For the function f whose graph is given, state the value of the given quantity, or say that it does not exist.



a) 3

b) 2

c) −2

d) DNE

e) 1

f) −1

g) −1

h) −1

i) −3

a) $\lim_{x\to 1} f(x)$ b) $\lim_{x\to 3^-} f(x)$ c) $\lim_{x\to 3^+} f(x)$ d) $\lim_{x\to 3} f(x)$ e) $f(3)$

f) $\lim_{x\to -2^-} f(x)$ g) $\lim_{x\to -2^+} f(x)$ h) $\lim_{x\to -2} f(x)$ i) $f(-2)$

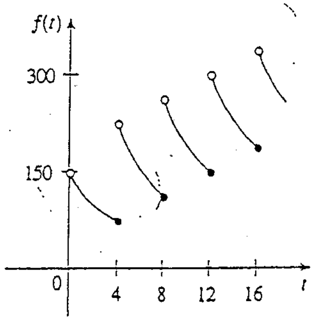
) Find $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$. HINT: Multiply top and bottom by $\sqrt{x+h}+\sqrt{x}$ (the conjugate of the numerator).

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

A patient receives a 150mg injection of a drug every 4 hours. The graph below shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find $\lim_{t\to 12^-} f(t)$ and $\lim_{t\to 12^+} f(t)$, and explain the significance of these one-sided limits. (That is, what do these limits say about the amount of the drug in the patient's bloodstream?)



$$\lim_{t\to 12^-} f(t) = 150, \quad \lim_{t\to 12^+} f(t) = 300$$

Right before the 12-hr. injection, the concentration has dropped down to 150. Right after the injection, it jumps up to 300.