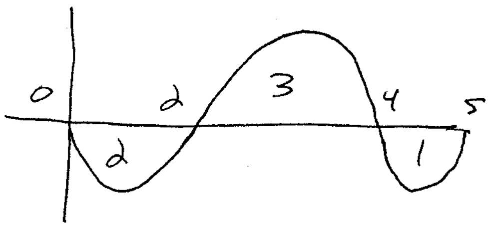
Math 5 Calculus Wiseman 11/27/02

Integral Worksheet

(1) The graph of $f(x)$ is pictured below. Find $\int_0^5 f(x)\,dx$, $\int_2^4 f(x)\,dx$, $\int_5^4 f(x)\,dx$, $\int_2^5 |f(x)|\,dx$ and $\int_0^5 -f(x)\,dx$.



$$\int_0^5 f(x)dx = -2+3-1=0$$

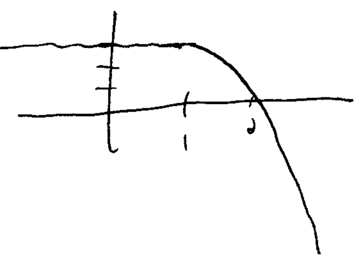
$$\int_2^4 f(x)\,dx = 3 \qquad \int_5^4 f(x)dx = -\int_4^5 f(x)dx = -(-1) =$$

$$\int_2^5 |f(x)|\,dx = 3+1=4$$

$$\int_0^5 -f(x)\,dx = -\int_0^5 f(x)\,dx = -0 = 0$$

(2) Find $\int_0^2 g(x)\,dx$, where g is defined by

$$g(x) = \begin{cases} 3, & \text{if } x \le 1 \\ 4 - x^2, & \text{if } x \ge 1. \end{cases}$$



$$\int_0^2 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx$$

$$= \int_0^1 3\,dx + \int_1^2 (4-x^2)\,dx$$

$$= 3x\Big|_0^1 + \left(4x - \frac{x^3}{3}\right)\Big|_1^2$$

$$= 3(1) - 3(0) + \left(4\cdot 2 - \frac{8}{3}\right) - \left(4\cdot 1 - \frac{1}{3}\right)$$

$$= 3 + 8 - \frac{8}{3} - 4 + \frac{1}{3} = 7 - 7/3 = 5 - 1/3$$

$$= 4\,2/3$$

(3) The linear density of a rod of length 4m is given by $\rho(x) = (9 + 2\sqrt{x})$ kg/m, where x is measured in meters from one end of the rod. Find the total mass of the rod.

$$\text{mass} = \int_0^4 \rho(x)\,dx \quad (\text{units are } kg/m \cdot m = kg \checkmark)$$

$$= \int_0^4 (9+2\sqrt{x})\,dx = \int_0^4 (9+2x^{\frac{1}{2}})\,dx$$

$$= \left(9x + 2\cdot\tfrac{2}{3}x^{\frac{3}{2}}\right)\Big|_0^4 = 9\cdot 4 + \tfrac{4}{3}4^{3/2} - (9\cdot 0 + \tfrac{4}{3}\cdot 0)$$

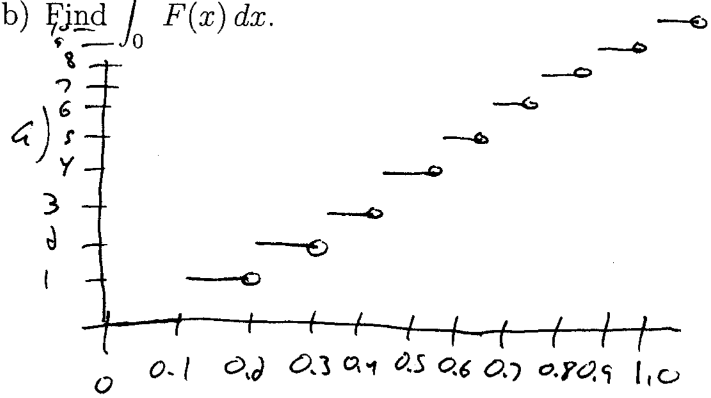
$$= 36 + \tfrac{4}{3}\cdot 8 = 36 + \tfrac{32}{3}$$

$$= 36 + 10 + 2/3 = 46\ 2/3\ kg$$

(4) Let $F(x)$ be defined on the interval $[0, 1]$ by the following rule: $F(x)$ is the first digit of the decimal expansion for x. For example, $F(1/3) = F(0.333\ldots) = 3$, and $F(0.721) = 7$.

(a) Sketch the graph of f, being sure to label the scale of each axis.

(b) Find $\int_0^1 F(x)\,dx$.



$$b) = 0\cdot 0.1 + 1\cdot 0.1 + 2\cdot 0.1 + \cdots + 9\cdot 0.1$$

$$= 0.1\,(1 + \cdots + 9) = 0.1\cdot\frac{9\cdot 10}{2} = 0.1\cdot 45 = 4.5$$