1. (25 pts) Determine whether the following series converge or diverge. Be sure to explain your reasoning.

(a) $\sum \frac{n!}{(n+2)!}$ $= \sum \frac{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{(n+2)(n+1)(n)(n-1)\cdots 3\cdot 2\cdot 1}$

$= \sum \frac{1}{(n+2)(n+1)}$ converges, by comparison to $\sum \frac{1}{n^2}$

(b) $\sum \frac{2^n}{n^n}$ Root test: $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^n}} = \lim_{n\to\infty} \frac{2}{n} = 0$

converges

(c) $\sum n(\frac{7}{8})^n$ Ratio test: $\lim_{n\to\infty} \frac{(n+1)\left(\frac{7}{8}\right)^{n+1}}{n\left(\frac{7}{8}\right)^n} = \lim_{n\to\infty} \frac{n+1}{n}\cdot\frac{7}{8}$

$= 1\cdot\frac{7}{8} < 1$

converges

(d) $\sum \frac{e^{\sin\frac{1}{n}}}{\cos\frac{1}{n}}$ Limit of the n^{th} term is $\lim_{n\to\infty} \frac{e^{\sin\frac{1}{n}}}{\cos\frac{1}{n}} = \frac{e^{\sin 0}}{\cos 0}$

$= \frac{e^0}{1} = \frac{1}{1}$

The n^{th} terms does not go to 0, so the series diverges

2. (25 pts) A ticket to a Phillies game costs \$20. 40,000 people go to the first home game of the season, and after that, the attendance for each home game is 98% of the attendance of the previous home game.

(a) How many people attend the 50th home game of the season?

1^{st} game : 40,000

2^{nd} game : $(.98)\,40{,}000$

3^{rd} game : $(.98)^2\,40{,}000$

$\vdots$

50^{th} game : $(.98)^{49}\cdot 40{,}000$

(b) If there are 81 home games in the season, what is the total attendance for the season?

$$(\#\text{ at }1^{st}\text{ game}) + (\#\text{ at }2^{nd}\text{ game}) + \dots + (\#\text{ at }81^{st}\text{ game})$$

$$= 40{,}000 + (.98)\,40{,}000 + \dots + (.98)^{80}\,40{,}000$$

$$= 40{,}000\cdot\frac{1-(.98)^{81}}{1-.98}$$

3. (25 pts) Decide whether each of the following statements is true or false. If one is false, explain why or give an example showing that it's false. (If it's true, you don't have to say anything else.)

(a) The sequence of partial sums of the series $1 + \frac{1}{2} + \frac{1}{3} + \ldots$ has a finite limit.

False: the harmonic series diverges

(b) If $a_n = 0$ for $n > 3,912$, then $\sum_{n=1}^{\infty} |a_n|$ converges.

True: $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{3912} |a_n| + 0 + 0 + 0 + \ldots = \sum_{n=1}^{3912} |a_n|$

(c) If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then the series $a_1 - a_2 - a_3 + a_4 - a_5 - a_6 + \ldots$ converges.

True - absolute convergence implies convergence

(d) $P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3!}$ is the third-degree Taylor polynomial approximation to $\ln x$ at $x = 0$.

False: $\ln 0$ is undefined, so $\ln$ doesn't have a Taylor polynomial at $x = 0$.

4. (25 pts) If I approximate $\sum_{n=0}^{\infty} \frac{5}{3^n + \sqrt{n}}$ by $5 + \frac{5}{4} = 6.25$, what's the most I could be off by?

$$\sum_{n=0}^{\infty} \frac{5}{3^n+\sqrt{n}} = \frac{5}{1+0} + \frac{5}{3+1} + \frac{5}{3^2+\sqrt{2}} + \frac{5}{3^3+\sqrt{3}} + \cdots$$

So the error is $\frac{5}{3^2+\sqrt{2}} + \frac{5}{3^3+\sqrt{3}} + \cdots + \frac{5}{3^n+\sqrt{n}} + \cdots$

I don't know what the sum is, but I know it's less than

$$\frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \cdots = \frac{\frac{5}{3^2}}{1-\frac{1}{3}} = \frac{5}{9} \cdot \frac{1}{2/3}$$

$$= \frac{5}{9} \cdot \frac{3}{2}$$

$$= \frac{5}{6}$$

So the error is $< \frac{5}{6}$

(EXTRA CREDIT) Which are better, sequences or series? Justify your answer.