Math 6B Practice Midterm

(1) Determine if the following series converge or diverge. Be sure to give reasons!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^4 - 6n^3 + 13n}{n^5 + n^2 + 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^{n-1} + 1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(n!)(n!)}{(2n)!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{2}{n}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} a_n$, if the nth partial sum of this series is given by $s_n = \dfrac{n-1}{2n+1}$.

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}\, a_n$, if you know that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a positive series that converges.

(2) Compute the sum of the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ when $|r| < 1$ and prove that your sum formula is correct.

(3) **Without** using the p-test, prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

(4) Find positive numbers A and B such that

$$0 < A \leq \sum_{n=1}^{\infty} \frac{1}{n^3} \leq B.$$

(5) Determine the interval and radius of convergence of the following series. (Include endpoints!)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+4)^n}{2n+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n! x^{2n}}{3^n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-222)^n}{n^2+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{n! 2^n x^n}{(2n)!}$

(6) (a) Find the Maclaurin series for $f(x) = e^{x^2}$.

(b) Use this series to compute $f^{(10)}(0)$. (Hint: What is the coefficient of x^{10} in part(a)? What should the coefficient be according to the definition of a Maclaurin series?)

(7) By integrating the geometric series expansion of $\dfrac{1}{1+x^2}$, we get the following series for $\arctan x$:

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}.$$

What is the interval of convergence of this series? (Be sure to check endpoints....)

(8) Suppose that at the beginning of each hour, a patient is given an injection of a 300 mg dose of antibiotics. It is known that after one hour, 43% of this antibiotic leaves one's system. So, the total amount of the drug **in** the patient after one hour is $300 + 300(.57)$ mg.
   (a) How many mg of the drug are in the body after 24 hours?
   (b) It turns out that 700 mg is a lethal dose of this antibiotic. Will the patient ever have this much of the drug in his/her system?

(9) For each part, first determine whether or not a such a series exists. If one does exist, give an example of such a series and explain why it is a valid example. If one does not exist, explain why not.
   (a) an alternating series that does not converge
   (b) a geometric series that converges to 2
   (c) a divergent series whose terms go to zero
   (d) a convergent series whose terms do not go to zero
   (e) a convergent series whose terms go to zero

(10) Find an infinite series equal to $\sin 1$.

(11) (a) Compute $T_4(x)$ (the fourth-degree Taylor polynomial) for $f(x) = \cos x$ at $a = 0$.
   (b) Compute the accuracy of using $T_4(x)$ to approximate $\cos 1$. (*i.e.,* What is a bound on the error?)