Math 6D Practice Exam Solns

1. Look at visit of \( \frac{1}{30} \) under slope-3 tent map:

\[
\frac{1}{30} \rightarrow \frac{3}{10} \rightarrow \frac{9}{10} \rightarrow \frac{9}{10} \rightarrow \frac{9}{10} \rightarrow \ldots
\]

It does stay in \([0,1]\) forever, so it's in \(K\).

2. Recall that a set is compact if it can be put in 1-1 correspondence with a subset of the natural numbers \(\mathbb{N} = 1, 2, 3, \ldots\). If \(S \subseteq T\), and \(T\) is countable, then \(S\) can be put in 1-1 correspondence with the \(S\)'s sink of \(\mathbb{N}\) put corresponds to \(n\)th of \(T\) that are also in \(S\), so \(S\)'s countable.

3. Use boxes of side-length \(\frac{1}{3^n}\) - need \(8^n\) of them to cover. So the dimension is:

\[
\lim_{n \to \infty} \frac{\log_2 8^n}{\log_2 3^n} = \frac{\log_2 8}{\log_2 3} = \frac{3}{2}.
\]

4. It can't be chaotic because a fixed pt. \(3/5\) is \(s\) sink. That means that \(3/5\) is not a sensitive pt. It also means that be periodic pt. aren't dense (because pts. near \(3/5\) can't be periodic, since my set closer & closer to \(3/5\)) and that there can't be a dense orbit (because if orbit that comes near \(3/5\) stays near, so can't "fill up" the interval \([0,1]\)).

5) There's a period-3 pt (coming from the point \(LMRC, \ell, \delta\)), so there are points of all periods.
Yes, they are dense. One way to see this is to realize that the dyadic intervals are the set of points with a terminating binary expansion. Another way is to realize that the intervals $\left[ \frac{m}{2^n}, \frac{m+1}{2^n} \right]$ are so thin, so smaller.

See class notes.

Look at the orbit of 0 under the map $z \rightarrow z^2 + c$.

9) $0 \rightarrow \frac{1}{2} \rightarrow (\frac{1}{4})^{\frac{1}{2}} = 3/4 \rightarrow \frac{9}{16} + \frac{1}{2} = \frac{17}{16} \rightarrow 2^{3/6} \rightarrow \cdots \rightarrow \infty$, so $\frac{1}{2}$ is not in the Mandelbrot set.

b) $i \rightarrow (i)^2 + i = -1 + i \rightarrow (-1 + i)^2 + i = -i \rightarrow -1 + i \rightarrow -i \cdots$

eventually period-2, so $i$ is in the Mandelbrot set.