1. Let \( F(x) = x^2 + x \). Find and classify the fixed points. What are the possible long-term behaviors of points under \( F \)?

2. Find and classify the fixed points of the following functions.
   (a) \( F(x) = \frac{1}{x^2} \).
   (b) \( F(x) = 1 - x^2 \).
   (c) \( F(x) = x^2 - 1 \).

3. (a) Let \( F(x) = -x^3 \). Find and classify the fixed points, the period-two points, and the period-three points.
   (b) Same question, with \( F(x) = \frac{1}{2}x + \frac{1}{2} \).

4. Let \( f(x) = \sqrt{x} \). What are the roots? What happens if you use Newton’s method with initial guess \( x_0 = 1 \)?

5. Is it possible for a discrete dynamical system \( F \) to have exactly three points of (least) period two?

6. (a) Let \( G(x) = \sin x \). Clearly 0 is a fixed point for \( G \). Is it attracting, repelling, or neither?
   (b) For what values of the parameter \( c \) will the point 0 be an attracting fixed point for the function \( G_c(x) = c \sin x \)?

7. Let \( G : [0, 1] \to [0, 1] \) be the doubling map defined by
   \[
   G(x) = \begin{cases} 
   2x & \text{if } 0 \leq x < 1/2, \\
   2x - 1 & \text{if } 1/2 \leq x \leq 1.
   \end{cases}
   \]
   (a) Draw the graphs of \( G \), \( G^2 \), and \( G^k \).
   (b) Find and classify the fixed points and period-two points of \( G \).
   (c) What is the orbit of the point \( 1/10 \)?
   (d) (HARDER) Show that \( x \) is eventually periodic or eventually fixed if and only if \( x \) is rational (i.e., \( x = p/q \), where \( p \) and \( q \) are integers).
   (e) ASIDE: \( G \) is clearly not continuous as a function from the interval \([0, 1]\) to itself. However, if we glue together the points 0 and 1, we get a circle, and \( G \) is continuous as a function from this circle to itself. This is the same thing as saying that \( G(x) = 2x \mod 1 \) (i.e., \( G(x) \) is the fractional part of \( 2x \)), if that means anything to you. Anyway, you don’t need to know any of this for your exam.