1. Let \( f(x) \) be a differentiable function, and let \( N(x) \) be its Newton function.
   (a) Calculate \( N'(x) \).
   (b) Show that every point \( x_0 \) such that \( f(x_0) = 0 \) and \( f'(x_0) \neq 0 \) is an attracting fixed point for \( N(x) \).
       What does this say about how well Newton’s method works?
   (c) Recall from your exam that the Babylonian function \( B(x) \) for finding the square root of a positive number \( a \) is \( B(x) = \frac{x + \frac{a}{x}}{2} \). Show that \( B(x) \) is the same as the Newton function for \( f(x) = x^2 - a \).

2. Recall from the practice midterm the doubling map \( G : [0, 1] \to [0, 1] \) defined by
   \[
   G(x) = \begin{cases} 
   2x & \text{if } 0 \leq x < 1/2, \\
   2x - 1 & \text{if } 1/2 \leq x \leq 1.
   \end{cases}
   \]
   (We can also define \( G \) as \( G(x) = 2x \mod 1 \).) Let \( L \) denote the subinterval \([0, 1/2]\) and \( R \) the subinterval \([1/2, 1]\).
   (a) Calculate and sketch the intervals \( L_L, L_R, R_L, R_R \) and \( L_{LL}, L_{LR}, L_{RL}, L_{RR}, R_{LL}, R_{LR}, R_{RL}, R_{RR} \).
   What is the size of an interval whose name has \( k \) letters?
   (b) Draw the transition graph for \( G \).

3. Recall that the tent map \( T : [0, 1] \to [0, 1] \) is defined by
   \[
   T(x) = \begin{cases} 
   2x & \text{if } 0 \leq x \leq 1/2, \\
   2 - 2x & \text{if } 1/2 \leq x \leq 1.
   \end{cases}
   \]
   Repeat parts (a) and (b) of #2 for \( T \).

4. Define a function \( H : [0, 1] \to [0, 1] \) by setting
   \[
   H(x) = \begin{cases} 
   -x + 1/3 & \text{if } 0 \leq x \leq 1/3, \\
   3x - 1 & \text{if } 1/3 \leq x \leq 2/3, \\
   -2x + 7/3 & \text{if } 2/3 \leq x \leq 1.
   \end{cases}
   \]
   Let \( L \) denote the subinterval \([0, 1/3]\), \( M \) the subinterval \([1/3, 2/3]\), and \( R \) the subinterval \([2/3, 1]\).
   (a) Calculate and sketch the intervals \( L_L, L_M, L_R, M_L, M_M, M_R, R_L, R_M, R_R \). What can you say about the size of an interval whose name has \( k \) letters in it? (This is more complicated than it was in #2 and #3.)
   (b) Draw the transition graph for \( H \).