1. Let $f(x)$ be a function defined on the interval $[0, \pi]$ such that $\int_0^\pi |f(x)|^2 \, dx = 7$. Let $\sum_{n=1}^\infty A_n \sin nx$ be its Fourier sine series. Show that $\lim_{n \to \infty} A_n = 0$.

2. The electric potential $v(x, y)$ in the space $0 \leq x \leq 1$, $y \geq 0$, satisfies the differential equation $\Delta v = 0$. Find the formula for $v$ if the planes $x = 0$ and $x = 1$ are kept at zero potential and the plane $y = 0$ at the potential $f(x)$, if $v$ is to be bounded as $y$ goes to infinity.

3. A function $T(r, \theta)$ satisfying $\Delta T = 0$ is bounded within the circle $r = 4$ and has values $T(4, \theta) = \sin \theta - 3 \cos \theta + 5 \sin 4\theta$. Find $T$.

4. Let $\phi(x) = 2 - \frac{x}{5}$ for $0 \leq x \leq 10$, and let $\hat{\phi}(x)$ be its Fourier sine series on the interval $[0, 10]$.
   (a) Does $\hat{\phi}$ converge to $\phi$ uniformly on $(0,10)$? Pointwise? In $L^2$?
   (b) For each $x$ ($0 \leq x \leq 10$), what is the sum of the series $\hat{\phi}(x)$?

5. Consider the problem of heat conduction in a uniform rectangular plate with no internal source, perfect lateral insulation, a prescribed temperature of $0^\circ$ on the sides $x = 0$ and $x = L$, and perfect insulation on the sides $y = 0$ and $y = H$. This problem is modeled mathematically as $u_t = k(uxx + uyy)$, subject to the boundary conditions $u(0, y, t) = u(L, y, t) = u_y(x, 0, t) = u_y(x, H, t) = 0$ and the initial condition $u(x, y, 0) = f(x, y)$. Find a formula for the general solution. (Be sure to explain how to find any constants that may appear in your solution.)

6. Suppose that $u(x, y)$ is harmonic on the unit disk $x^2 + y^2 < 1$, and also satisfies the boundary conditions $u(x, y) = 10$ on the part of the boundary of the disk that lies in the first quadrant (i.e., in polar coordinates, the set $r = 1$, $0 \leq \theta \leq \pi/2$), and $u(x, y) = 0$ on the rest of the boundary of the disk (i.e., $r = 1$, $\pi/2 < \theta < 2\pi$). What is $u(0, 0)$?

7. Give an example of a bounded set $D$ in the plane and a function $u(x, y)$ that is harmonic in the interior of $D$ and continuous on $\overline{D} = D \cup (\text{bdy } D)$, such that the maximum value of $u$ is attained in the interior of $D$, but $u$ is not a constant function. Why doesn’t your example violate the maximum principle?